1. Define link, Inversion, Kinematic pair, Kinematic chain, Machine
2. List inversions of a four bar chain and explain Whitworth quick return motion mechanism
3. Explain Pantograph
4. Explain Ackerman steering gear mechanism with a neat sketch
5. Explain ratchet and pawl mechanism
6. Explain drag link mechanism
7. Determine the degrees of freedom for the following links
8. With a neat sketch derive the Length of arc of contact Derivation
9. Two gears in mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 teeth while the pinion has 23 teeth. If the addendum on pinion and gear wheel are equal to one module, find
   a) The number of pairs of teeth in contact
   b) The angle of action of the pinion and the gear wheel
10. With a neat sketch list the classification of followers
11. Two gear wheel mesh externally and are to give a velocity ratio of 3. The teeth are of involute form of module 6 mm and standard addendum one module. Pressure angle=18°. Pinion rotates at 90 rpm. Find
   a) Number of teeth on each wheel so that interference is just avoided
   b) Length of path of contact
   c) Length of arc of contact
   d) Maximum velocity of sliding between teeth
12. Explain law of Gearing with a neat sketch
13. Define Train value and velocity ratio and explain different methods to find velocity ratio or train value?
14. In an epicyclic gear train as shown in the fig 1. the arm A is fixed to the shaft ‘S’.
   The wheel ‘B’ having 100 teeth rotates freely on this shaft ‘S’ wheel ‘F’ 150 teeth
separately driven. If the arm A runs at 200 rpm, wheel F at 100 rpm in the same direction find (1) No of teeth of gear C (2)Speed of wheel ‘B’

15. The gear train shown in the fig 2 Gear A Meshes with gear B. In the compound gear B-C, Gear C meshes with gear D, rotating relative to A around the same axis of A. If the gear A is fixed arm F is used as the driving member, determine the speed ratio ND/NF. Number of teeth on wheels A, B, C and D are 61, 61, 62, 60 respectively.

16. List different types of quick return motion mechanisms and explain Drag link mechanism?

17. Sketch Peaucellier mechanism and Roberts mechanism?

18. Explain Geneva mechanism with a neat sketch?

19. Explain condition for correct steering in Motor car?

20. An epicyclic gear train is shown in fig 3. The wheel A is fixed and the input at the arm R is 3kw at 600 rpm. Find the speed of wheel D and the torque on it and the torque required to hold the wheel ‘A’ neglect frictional losses.

21. Explain torque in an epicyclic gear train?

22. The fig 4 shows an epicyclic gear train. Wheel E is fixed and wheels C and D are integrally cast and mount on the same pin. If arm A makes one revolution per sec counterclockwise determine the speed and direction of rotation of the wheels B and F

23. A cam rotating clockwise at uniform speed of 300 rpm operates a reciprocating follower through a roller 1.5cm diameter. The follower motion is defined as below
   a. Outward during 150° with UARM
   b. Dwell for next 30°
   c. Return during next 120° with SHM
   d. Dwell for the remaining period

   Stroke of the follower is 3cm. minimum radius of the cam is 3cm. Draw the cam profile the follower axis passes through the cam.

24. Define Gear train and explain different types of gear train

25. In an epicyclic gear train, the internal wheels A, B and the compound wheel C and D rotate independently about the axis ‘O’. The wheels E and F rotate on a pin fixed to the arm G. E gears with A and C, and F gears with B and D. All the wheels have
same pitch and the number of teeth on E and F are 18, c=28, D=26

1. Sketch the arrangement
2. Find the number of teeth on A and B
3. If the arm G makes 150 rpm CW and A is fixed, find speed of B
4. If the arm G makes 150 rpm CW and wheel A makes 15 rpm CCW, find the speed of B.

26. An epicyclic gear train is shown in fig the number of teeth on wheel A, B and C are 48, 24 and 50 respectively. If the arm rotates 400rpm clockwise find
   a. Speed of wheel C when A is fixed
   b. Speed of wheel A when C is fixed
CHAPTER – 1

Quick return motion mechanisms.
Quick return mechanisms are used in machine tools such as shapers and power driven saws for the purpose of giving the reciprocating cutting tool a slow cutting stroke and a quick return stroke with a constant angular velocity of the driving crank. Some of the common types of quick return motion mechanisms are discussed below. The ratio of time required for the cutting stroke to the time required for the return stroke is called the time ratio and is greater than unity.

Drag link mechanism
This is one of the inversions of four bar mechanism, with four turning pairs. Here, link 2 is the input link, moving with constant angular velocity in anti-clockwise direction. Point C of the mechanism is connected to the tool post E of the machine. During cutting stroke, tool post moves from E1 to E2. The corresponding positions of C are C1 and C2 as shown in the fig. For the point C to move from C1 to C2, point B moves from B1 to B2, in anti-clockwise direction. IE, cutting stroke takes place when input link moves through angle B1AB2 in anti-clockwise direction and return stroke takes place when input link.

The time ratio is given by the following equation.

Whitworth quick return motion mechanism:
This is first inversion of slider mechanism, where, crank 1 is fixed. Input is given to link 2, which moves at constant speed. Point C of the mechanism is connected to the tool post.
D of the machine. During cutting stroke, tool post moves from D1 to D11. The corresponding positions of C are C1 and C11 as shown in the fig. 1.38. For the point C to move from C1 to C11, point B moves from B1 to B11, in anti-clockwise direction. I.E., cutting stroke takes place when input link moves through angle B1O2B11 in anticlockwise direction and return stroke takes place when input link moves through angle B11O2B1 in anti-clockwise direction.

Crank and slotted lever quick return motion mechanism
This is second inversion of slider mechanism, where, connecting rod is fixed. Input is given to link 2, which moves at constant speed. Point C of the mechanism is connected to the tool post D of the machine. During cutting stroke, tool post moves from D1 to D11. The corresponding positions of C are C1 and C11 as shown in the fig. 1.39. For the point C to move from C1 to C11, point B moves from B1 to B11, in anti-clockwise direction. I.E., cutting stroke takes place when input link moves through angle B1O2B11 in anticlockwise direction and return stroke takes place when input link moves through angle B11O2B1 in anti-clockwise direction.
**Straight line motion mechanisms**

Straight line motion mechanisms are mechanisms, having a point that moves along a straight line, or nearly along a straight line, without being guided by a plane surface.

**Condition for exact straight line motion:**

If point B (fig.1.40) moves on the circumference of a circle with center O and radius OA, then, point C, which is an extension of AB traces a straight line perpendicular to AO, provided product of AB and AC is constant.
Locus of pt. C will be a straight line, ⊥ to AE if, is constant

**Proof:**

**Peaucellier exact straight line motion mechanism:**

Fig. Here, AE is the input link and point E moves along a circular path of radius AE = AB. Also, EC = ED = PC = PD and BC = BD. Point P of the mechanism moves along exact straight line, perpendicular to BA extended.

To prove B, E and P lie on same straight line:

Triangles BCD, ECD and PCD are all isosceles triangles having common base CD and apex points being B, E and P. Therefore points B, E and P always lie on the perpendicular bisector of CD. Hence these three points always lie on the same straight line.

To prove product of BE and BP is constant.

In triangles BFC and PFC,

\[ BC^2 = FB^2 + FC^2 \] and \[ PC^2 = PF^2 + FC^2 \]

\[ \therefore BC^2 - PC^2 = FB^2 - PF^2 = (FB + PF)(FB - PF) = BP \times BE \]

But since BC and PC are constants, product of BP and BE is constant, which is the condition for exact straight line motion. Thus point P always moves along a straight line perpendicular to BA as shown in the fig. 1.41.
**Approximate straight line motion mechanism:** A few four bar mechanisms with certain modifications provide approximate straight line motions.

**Robert’s mechanism**

![Robert’s mechanism diagram](image)

This is a four bar mechanism, where, PCD is a single integral link. Also, dimensions AC, BD, CP and PD are all equal. Point P of the mechanism moves very nearly along line AB.

**Intermittent motion mechanisms**

An intermittent-motion mechanism is a linkage which converts continuous motion into intermittent motion. These mechanisms are commonly used for indexing in machine tools.

**Geneva wheel mechanism**

![Geneva wheel mechanism diagram](image)
In the mechanism shown (Fig.1.43), link A is driver and it contains a pin which engages with the slots in the driven link B. The slots are positioned in such a manner, that the pin enters and leaves them tangentially avoiding impact loading during transmission of motion. In the mechanism shown, the driven member makes one-fourth of a revolution for each revolution of the driver. The locking plate, which is mounted on the driver, prevents the driven member from rotating except during the indexing period.

**Ratchet and pawl mechanism**

![Diagram of ratchet and pawl mechanism](image)

Ratchets are used to transform motion of rotation or translation into intermittent rotation or translation. In the fig.1.44, A is the ratchet wheel and C is the pawl. As lever B is made to oscillate, the ratchet wheel will rotate anticlockwise with an intermittent motion. A holding pawl D is provided to prevent the reverse motion of ratchet wheel.

**Other mechanisms**

**Toggle mechanism**
Toggle mechanisms are used, where large resistances are to be overcome through short distances. Here, effort applied will be small but acts over large distance. In the mechanism shown in fig.1.45, 2 is the input link, to which, power is supplied and 6 is the output link, which has to overcome external resistance. Links 4 and 5 are of equal length. Considering the equilibrium condition of slider 6, for small angles of \( \alpha \), F (effort) is much smaller than P(resistance). This mechanism is used in rock crushers, presses, riveting machines etc.

**Pantograph**

Pantographs are used for reducing or enlarging drawings and maps. They are also used for guiding cutting tools or torches to fabricate complicated shapes.

In the mechanism shown in fig.1.46 path traced by point A will be magnified by point E to scale, as discussed below. In the mechanism shown, AB = CD; AD = BC and OAE lie on a straight line. When point A moves to \( A' \), E moves to \( E' \) and \( OA'E' \) also lies on a straight line. From the fig.1.46, \( \Delta ODA \equiv \Delta OCE \) and \( \Delta OD'A' \equiv \Delta OC'E' \).

**Hooke’s joint (Universal joints)**

Hooke’s joins is used to connect two nonparallel but intersecting shafts. In its basic
shape, it has two U-shaped yokes ‘a’ and ‘b’ and a center block or cross-shaped piece, C. fig The universal joint can transmit power between two shafts intersecting at around 300 angles (α). However, the angular velocity ratio is not uniform during the cycle of operation. The amount of fluctuation depends on the angle (α) between the two shafts.
For uniform transmission of motion, a pair of universal joints should be used fig.
Intermediate shaft 3 connects input shaft 1 and output shaft 2 with two universal joints. The angle α between 1 and 2 is equal to angle α between 2 and 3. When shaft 1 has uniform rotation, shaft 3 varies in speed; however, this variation is compensated by the universal joint between shafts 2 and 3. One of the important applications of universal joint is in automobiles, where it is used to transmit power from engine to the wheel axle.

Steering gear mechanism
The steering mechanism is used in automobiles for changing the directions of the wheel axles with reference to the chassis, so as to move the automobile in the desired path. Usually, the two back wheels will have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of front wheels.
In automobiles, the front wheels are placed over the front axles (stub axles), which are pivoted at the points A & B as shown in the fig.1.48. When the vehicle takes a turn, the front wheels, along with the stub axles turn about the pivoted points. The back axle and the back wheels remain straight. Always there should be absolute rolling contact between the wheels and the road surface. Any sliding motion will cause wear of tyres. When a vehicle is taking turn, absolute rolling motion of the wheels on the road surface possible, only if all the wheels describe concentric circles. Therefore, the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheel.

**Condition for perfect steering**

The condition for perfect steering is that all the four wheels must turn about the same instantaneous centre. While negotiating a curve, the inner wheel makes a larger turning angle \( \theta \) than the angle \( \phi \) subtended by the axis of the outer wheel.

In the fig. \( a = \) wheel track, \( L = \) wheel base, \( w = \) distance between the pivots of front axles.
This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels when the vehicle takes a turn.

**Ackermann steering gear mechanism**

[Diagram of Ackermann steering mechanism]

Ackerman steering mechanism, RSAB is a four bar chain as shown in fig.1.50. Links RA and SB which are equal in length are integral with the stub axles. These links are connected with each other through track rod AB. When the vehicle is in straight ahead position, links RA and SB make equal angles $\alpha$ with the center line of the vehicle. The dotted lines in fig.1.50 indicate the position of the mechanism when the vehicle is turning left. Angle $\alpha$ can be determined using the above equation. The values of $\theta$ and $\varphi$ to be taken in this equation are those found for correct steering using the equation $L$

This mechanism gives correct steering in only three positions. One, when $\theta = 0$ and other two each corresponding to the turn to right or left (at a fixed turning angle, as determined by equation). The correct values of $\varphi$, $[\varphi_c]$ corresponding to different values of $\theta$, for correct steering can be determined using equation [2]. For the given dimensions of the mechanism,
actual values of $\varphi$, [$\varphi_a$] can be obtained for different values of $\theta$. The difference between $\varphi_c$ and $\varphi_a$ will be very small for small angles of $\theta$, but the difference will be substantial, for larger values of $\theta$. Such a difference will reduce the life of tyres because of greater wear on account of slipping.

But for larger values of $\theta$, the automobile must take a sharp turn; hence it will be moving at a slow speed. At low speeds, wear of the tyres is less. Therefore, the greater difference between $\varphi_c$ and $\varphi_a$ larger values of $\theta$ ill not matter.

As this mechanism employs only turning pairs, friction and wear in the mechanism will be less. Hence its maintenance will be easier and is commonly employed in automobiles.
CHAPTER 2 and 3

VELOCITY AND ACCELERATION

• Introduction

Kinematics deals with study of relative motion between the various parts of the machines. Kinematics does not involve study of forces. Thus motion leads study of displacement, velocity and acceleration of a part of the machine. Study of Motions of various parts of a machine is important for determining their velocities and accelerations at different moments. As dynamic forces are a function of acceleration and acceleration is a function of velocities, study of velocity and acceleration will be useful in the design of mechanism of a machine. The mechanism will be represented by a line diagram which is known as configuration diagram. The analysis can be carried out both by graphical method as well as analytical method.

• Some important Definitions

Displacement: All particles of a body move in parallel planes and travel by same distance is known, linear displacement and is denoted by ‘x’.
A body rotating about a fired point in such a way that all particular move in circular path angular displacement
Velocity: Rate of change of displacement is velocity. Velocity can be linear velocity of angular velocity. Relation between linear velocity and angular velocity.

Some important Definitions

Displacement: All particles of a body move in parallel planes and travel by same distance is known, linear displacement and is denoted by ‘x’.
A body rotating about a fired point in such a way that all particular move in circular path angular displacement and is denoted by ‘θ’.

Velocity: Rate of change of displacement is velocity. Velocity can be linear velocity of angular velocity.Linear velocity is Rate of change of linear displacement,Angular velocity is Rate of change of angular displacement .Relation between linear velocity and angular velocity.

Acceleration: Rate of change of velocity

Absolute velocity: Velocity of a point with respect to a fixed point (zero velocity point).A
\[ V_a = \vec{d}_2 \times \vec{r} \]
\[ V_a = \vec{d}_2 \times \vec{O}_2 \vec{A} \]

Ex: \( V_{ao2} \) is absolute velocity.

**Relative velocity:** Velocity of a point with respect to another point ‘x’

\[ \text{Ex: } V_{ba} \text{ Velocity of point B with respect to A} \]

**Note:** Capital letters are used for configuration diagram. Small letters are used for velocity vector diagram.

This is absolute velocity

Velocity of point A with respect to \( O_2 \) fixed point, zero velocity point.

\[ V_{ba} = \text{ or } V_{ab} \]

Equal in magnitude but opposite in direction.

\[ V_b \text{ Absolute velocity is velocity of B with respect to } O_4 \text{ (fixed point, zero velocity point)} \]
Velocity vector diagram

Vector $O_2a = V_a$ = Absolute velocity
Vector $ab = V_{ab}$

$V_{ba}$ is equal magnitude with $V_{ba}$ but is opposite in direction.

Vector $O_4b = V_b$ Absolute velocity.

To illustrate the difference between absolute velocity and relative velocity. Let, us consider a simple situation.

A link AB moving in a vertical plane such that the link is inclined at 30° to the horizontal with point A is moving horizontally at 4 m/s and point B moving vertically upwards. Find velocity of B.

$V_a = 4 \text{ m/s}$ ab Absolute velocity Horizontal direction

$V_b = ?$ ab Absolute velocity Vertical direction
Velocity of B with respect to A is equal in magnitude to velocity of A with respect to B but opposite in direction. □ **Relative Velocity Equation**

1. By graphical method
2. By relative velocity method
3. By instantaneous method
By Graphical Method

The following points are to be considered while solving problems by this method.

1. Draw the configuration design to a suitable scale.
2. Locate all fixed point in a mechanism as a common point in velocity diagram.
3. Choose a suitable scale for the vector diagram velocity.
4. The velocity vector of each rotating link is $\vec{v}$ to the link.
5. Velocity of each link in mechanism has both magnitude and direction. Start from a point whose magnitude and direction is known.
6. The points of the velocity diagram are indicated by small letters.

To explain the method let us take a few specific examples.

1. **Four – Bar Mechanism:** In a four bar chain ABCD link AD is fixed and in 15 cm long.
   
   The crank AB is 4 cm long rotates at 180 rpm (cw) while link CD rotates about D is 8 cm long BC = AD and $|BAD = 60^\circ$. Find angular velocity of link CD.

![Configuration Diagram](image)

**Velocity vector diagram**

\[
V_b = |\vec{r}| = |\vec{ba} \times AB| = \frac{2\pi \times 120}{60} \times 4 = 50.24 \text{ cm/sec}
\]
Choose a suitable scale

1 cm = 20 m/s = \( ab \)

\[ V_{cb} = bc \]
\[ V_c = dc = 38 \text{ cm/sec} = V_{cd} \]

We know that \( V = \omega R \)

\[ V_{cd} = \omega \text{CD} \times \text{CD} \]

\[ \omega_{cd} = \frac{V_{cd}}{\text{CD}} = 38 \times 4.75 \text{rad/sec (cw)} \]

CD 8

2. Slider Crank Mechanism:

In a crank and slotted lever mechanism crank rotates of 300 rpm in a counter clockwise direction. Find

(i) Angular velocity of connecting rod and (ii) Velocity of slider.

Configuration diagram

Step 1: Determine the magnitude and velocity of point A with respect to 0,

\[ V_A = \omega_{01A} \times O_2A = \frac{2\pi \times 300}{60 \times 60} = 600 \text{ mm/sec} \]
Step 2: Choose a suitable scale to draw velocity vector diagram.

![Velocity vector diagram]

3. Shaper Mechanism:

In a crank and slotted lever mechanisms crank $O_2A$ rotates at $\omega \text{ rad/sec}$ in CCW direction. Determine the velocity of slider.

![Configuration diagram]

Scale 1 cm = $\ldots \times \ldots \text{ m/s}$

![Velocity vector diagram]

Scale 1 cm = $\ldots \times \ldots \text{ m/s}$
To Determine Velocity of Rubbing

Two links of a mechanism having turning point will be connected by pins. When the links are motion they rub against pin surface. The velocity of rubbing of pins depends on the angular velocity of links relative to each other as well as direction.

For example: In a four bar mechanism we have pins at points A, B, C and D.

**Problem 1:**

In a four bar mechanism, the dimensions of the links are as given below:

\[
\begin{align*}
    AB & = 50 \text{ mm}, & BC & = 66 \text{ mm} \\
    CD & = 56 \text{ mm} & AD & = 100 \text{ mm}
\end{align*}
\]

At a given instant when \( \angle DAB \equiv 60^\circ \) the angular velocity of link AB is 10.5 rad/sec in CCW direction.

Determine,

i) Velocity of point C

ii) Velocity of point E on link BC when BE = 40 mm iii) The angular velocity of link BC and CD

iv) The velocity of an offset point F on link BC, if BF = 45 mm, CF = 30 mm and BCF is read clockwise.

v) The velocity of an offset point G on link CD, if CG = 24 mm, DG = 44 mm and DCG is read clockwise.

vi) The velocity of rubbing of pins A, B, C and D. The ratio of the pins are 30 mm, 40 mm, 25 mm and 35 mm respectively.

**Solution:**

**Step 1:** Construct the configuration diagram selecting a suitable scale.

Scale: 1 cm = 20 mm
Step – 2: Given the angular velocity of link AB and its direction of rotation determine velocity of point with respect to A (A is fixed hence, it is zero velocity point).

\[ V_{ba} = \mathbf{BA} \times \mathbf{BA} \]
\[ = 10.5 \times 0.05 = 0.525 \text{ m/s} \]

Step – 3: To draw velocity vector diagram choose a suitable scale, say 1 cm = 0.2 m/s. 

First locate zero velocity points.

- Draw a line \( r \) to link AB in the direction of rotation of link AB (CCW) equal to 0.525 m/s.

- From b draw a line \( r \) to BC and from d. Draw d line \( r \) to CD to intersect at C.

- \( V_{cb} \) is given vector bc \( V_{bc} = 0.44 \text{ m/s} \)

- \( V_{cd} \) is given vector dc \( V_{cd} = 0.39 \text{ m/s} \)

Step – 4: To determine velocity of point E (Absolute velocity) on link BC, first locate the position of point E on velocity vector diagram. This can be done by taking corresponding ratios of lengths of links to vector distance i.e.

\[ be = \frac{BE}{bc} \]
\[ \frac{0.04}{BE} \times V_{cb} = \frac{0.04}{0.44} \times 0.44 = 0.24 \text{ m/s} \]
Join e on velocity vector diagram to zero velocity points a, d / vector de = \( V_e = 0.415 \) m/s.

Step 5: To determine angular velocity of links BC and CD, we know \( V_{bc} \) and \( V_{cd} \).

\[
\begin{align*}
\omega_{BC} &= \frac{V_{bc}}{0.44} \text{ r/s (cw) BC} \\
\omega_{CD} &= \frac{V_{cd}}{0.39} \text{ r/s (CCW) CD}
\end{align*}
\]

Similarly,

\[
\begin{align*}
V_{cd} &= \omega_{CD} \times CD \\
V_{bc} &= \omega_{BC} \times BC
\end{align*}
\]

Step 6: To determine velocity of an offset point F

- Draw a line \( \overline{CF} \) to CF from C on velocity vector diagram.
- Draw a line \( \overline{BF} \) to BF from b on velocity vector diagram to intersect the previously drawn line at ‘f’.
- From the point f to zero velocity point a, d and measure vector \( \overline{fa} \) to get \( V_f = 0.495 \) m/s.

Step 7: To determine velocity of an offset point.

- Draw a line \( \overline{GC} \) to GC from C on velocity vector diagram.
- Draw a line \( \overline{DG} \) to DG from d on velocity vector diagram to intersect previously drawn line at g.
- Measure vector \( \overline{dg} \) to get velocity of point G.

\( V_g = \overline{dg} = 0.305 \) m/s

Step 8: To determine rubbing velocity at pins

- Rubbing velocity at pin A will be
  \( V_{pa} = \overline{ab} \times r \) of pin A \( V_{pa} = 10.5 \times 0.03 \)
  \[= 0.315 \text{ m/s} \]

- Rubbing velocity at pin B will be
  \( V_{pb} = (\overline{ab} + \overline{cb}) \times r_{pb} \) of point at B.
  \[
  \begin{align*}
  &\text{[\overline{ab} CCW and \overline{cb} CW]} \\
  &\overline{ab} = (10.5 + 6.6) \times 0.04 = 0.684 \text{ m/s.}
  \end{align*}
  \]
Rubbing velocity at point C will be
Problem 2:

In a slider crank mechanism the crank is 200 mm long and rotates at 40 rad/sec in a CCW direction. The length of the connecting rod is 800 mm. When the crank turns through 60° from Inner-dead centre.

Determine,

i) The velocity of the slider

ii) Velocity of point E located at a distance of 200 mm on the connecting rod extended.

iii) The position and velocity of point F on the connecting rod having the least absolute velocity.

iv) The angular velocity of connecting rod.

v) The velocity of rubbing of pins of crank shaft, crank and cross head having pins diameters 80, 60 and 100 mm respectively.

Solution:

Step 1: Draw the configuration diagram by selecting a suitable scale.

\[ \begin{align*}
V_a &= W_{oa} \times OA \\
V_a &= 40 \times 0.2 \\
V_a &= 8 \text{ m/s}
\end{align*} \]

Step 2: Choose a suitable scale for velocity vector diagram and draw the velocity vector diagram.

- Mark zero velocity point o, g.

- Draw \( \overrightarrow{oa} \) to link OA equal to 8 m/s
• From a draw a line $\overrightarrow{ef}$ to AB and from o, g draw a horizontal line (representing the line of motion of slider B) to intersect the previously drawn line at b.

• ab give $V_{ba}=4.8$ m/sec

**Step 3**: To mark point 'e' since 'E' is on the extension of link AB drawn be =

$BE \overrightarrow{AB}$

$\overrightarrow{xe}$ ab mark the point e on extension of vector ba. Join e to o, g. ge will give velocity of point E.

$V_e = ge=8.4$ m/sec

Step 4: To mark point F on link AB such that this has least velocity (absolute).

Draw a line $\overrightarrow{ef}$ to ab passing through o, g to cut the vector ab at f. From f to

o, g. gf will have the least absolute velocity.

To mark the position of F on link AB.

Find BF by using the relation.

$\overrightarrow{fb} \overrightarrow{ab}$

$\overrightarrow{BF} \overrightarrow{AB}$

$BF \overrightarrow{ab} \times AB = 200$ mm $\overrightarrow{ab}$

**Step 5**: To determine the angular velocity of connecting rod.

We know that $V_{ab} = \overrightarrow{ab} \times AB$

$\overrightarrow{ab} = \overrightarrow{v_{ab}} = 6$ rad/sec $\overrightarrow{AB}$

**Step 6**: To determine velocity of rubbing of pins.

• $V_{pcrankshaft} = \overrightarrow{ao} \times$ radius of crankshaft pin
\[ V_b = 4.18 \times 0.3 = 1.254 \text{ m/sec} \]

**Step 3:** Draw velocity vector diagram.
Draw \( O_1b \) to link \( O_1B \) equal to 1.254 m/s. From \( b \) draw a line along the line of \( O_2B \) and from \( O_1O_2 \) draw a line to \( O_2B \). This intersects at \( c \). \( c \) will measure velocity of sliding of slider and \( O_2C \) will measure the velocity of \( C \) on link \( O_2C \).

Since point \( D \) is on the extension of link \( O_2C \) measure \( O_2d \) such that From \( d \) draw a line to link \( DR \) and from \( O_1O_2 \). Draw a line along the line of stroke of Ram R (horizontal). These two lines will intersect at point \( r \). \( O_2r \) will give the velocity of Ram R.

To determine the angular velocity of link \( O_2D \) determine \( V_d = O_2d \).

• Problem 4: Figure below shows a toggle mechanisms in which the crank \( OA \) rotates at 120 rpm. Find the velocity and acceleration of the slider \( D \).

• Solution:

All the dimensions in mm

Configuration Diagram

Step 1: Draw the configuration diagram choosing a suitable scal.
Step 2: Determine velocity of point A with respect to O.

\[
V_{ao} = \square OA \times OA \\
2 \square \times 120 \\
V_{ao} = \frac{x}{0.4} \square 0.4 \square 5.024 \text{ m/s}
\]

Step 3: Draw the velocity vector diagram.

Choose a suitable scale
Mark zero velocity points O,q

→

Draw vector \(oa\) \(\square\) to link OA and magnitude = 5.024 m/s.

**Velocity vector diagram**

- From a draw a line \(\square\) to AB and from q draw a line \(\square\) to QB to intersect at b.
  \(ab\) \(\square\) \(V_{ba}\) and \(qb\) \(\square\) \(V_{bq}\).
- Draw a line \(\square\) to BD from b from q draw a line along the slide to intersect at d.
  \(dq\) \(\square\) \(V_{d}\) (slider velocity)

**Problem 5:** A whitworth quick return mechanism shown in figure has the following dimensions of the links.

The crank rotates at an angular velocity of 2.5 r/s at the moment when crank makes an angle of 45° with vertical. Calculate

a) the velocity of the Ram S
b) the velocity of slider P on the slotted level
c) the angular velocity of the link RS.

**Solution:**

Step 1: To draw configuration diagram to a suitable scale.
Step 2: To determine the absolute velocity of point P.

\[ V_P = |\overrightarrow{OP}| \times OP \]

\[ 2 \times 240 \]

\[ V_{ao} = \frac{OP}{60} \times 0.24 \times 0.6 \text{ m/s} \]

Step 3: Draw the velocity vector diagram by choosing a suitable scale.

- Draw \( \overrightarrow{OP} \) link \( OP = 0.6 \) m.
From O, draw a line to AP/AQ and from P draw a line along AP to intersect previously drawn line at Q. \( R_P \) = Velocity of sliding.

\[ V_{aq} = \text{Velocity of Q with respect to A.} \]

Angular velocity of link RS = \( \frac{SR}{RS} \) rad/sec

Problem 6: A toggle mechanism is shown in the figure along with the diagrams of the links in mm. Find the velocities of the points B and C and the angular velocities of links AB, BQ and BC. The crank rotates at 50 rpm in the clockwise direction.

Solution

Step 1: Draw the configuration diagram to a suitable scale.

Step 2: Calculate the magnitude of velocity of A with respect to O.

\[ V_a = OA \times \omega \]

\[ V_a = 30 \times 50 \times 0.03 \times 0.05 \times 0.1507 \, \text{m/s} \]

All dimensions are in mm.

OA = 30
AB = 80
BQ = 100
BC = 100
Step 3: Draw the velocity vector diagram by choosing a suitable scale.

- Draw $Oa$ to link $OA = 0.15\ m/s$
- From $a$ draw a link to $AB$ and from $O$, $q$ draw a link to $BQ$ to intersect at $b$.

$$\overrightarrow{ab} = a_{ba}$$
$$\overrightarrow{qb} = V_{bq}$$
$$a_{ab} = 0.74r/\text{s (ccw)}$$
$$b_{qb} = 1.3r/\text{s (ccw)}$$

- From $b$ draw a line to $Be$ and from $O$, these two lines intersect at $C$.
$$OC = V_C$$
$$b_{C} = V_{Cb}$$
$$b_{BC} = 1.33r/\text{s (ccw)}$$

**Problem 7:** The mechanism of a stone crusher has the dimensions as shown in figure in mm. If crank rotates at $120\ rpm\ CW$. Find the velocity of point $K$ when crank $OA$ is inclined at $30^\circ$ to the horizontal. What will be the torque required at the crank to overcome a horizontal force of $40\ kN$ at $K$. 

Vector velocity diagram
Step 1: Draw the configuration diagram to a suitable scale.

Step 2: Given speed of crank OA determine velocity of A with respect to ‘o’.

\[ V_a = \omega \cdot OA \times OA = \omega \cdot 2 \times 120 \times 0.1 \times 60 = 1.26 \text{ m/s} \]

Step 3: Draw the velocity vector diagram by selecting a suitable scale.
Draw Oa \( \overrightarrow{A} \) to link OA = 1.26 m/s. From a draw a link \( \overrightarrow{B} \) to AB and from q draw a link \( \overrightarrow{B} \) to BQ to intersect at b. From b draw a line \( \overrightarrow{C} \) to BC and from a, draw a line \( \overrightarrow{C} \) to AC to intersect at c. From c draw a line \( \overrightarrow{D} \) to CD and from m draw a line \( \overrightarrow{D} \) to MD to intersect at d.

From d draw a line \( \overrightarrow{E} \) to KD and from m draw a line \( \overrightarrow{E} \) to KM to x intersect the previously drawn line at k.

Since we have to determine the torque required at OA to overcome a horizontal force of 40 kN at K. Draw a the horizontal line from o, q, m and c line \( \overrightarrow{F} \) to this line from k.

\[
\begin{align*}
\mathbf{V} &= \mathbf{r} \\
\mathbf{T} &= \mathbf{F} \times \mathbf{P} \\
\mathbf{F} &= \mathbf{T} \mathbf{OA}
\end{align*}
\]

\[\mathbf{T}_{OA} = \mathbf{F}_k \mathbf{V}_h \text{ horizontal}\]

Problem 8: In the mechanism shown in figure link OA = 320 mm, AC = 680 mm, \( \mathbf{OA} \), and OQ = 650 mm.

Determine,

i) The angular velocity of the cylinder

ii) The sliding velocity of the plunger

iii) The absolute velocity of the plunger

When the crank OA rotates at 20 rad/sec clockwise.

* Solution:

Step 1: Draw the configuration diagram.

Step 2: Draw the velocity vector diagram o Determine velocity of point A with respect to O.
$V_a = \vec{OA} \times OA = 20 \times 0.32 = 6.4 \text{ m/s}$

- Select a suitable scale to draw the velocity vector diagram.

- Mark the zero velocity point. Draw vector $\vec{oa}$ to link $OA$ equal to 6.4 m/s.

- From a draw a line $\vec{a}f$ to AB and from o, q, draw a line perpendicular to AB. To mark point c on $\vec{ab}$

  We know that $\vec{ab} \cdot \vec{AB} = AC$

  $\vec{ac} \cdot \vec{ab} \times AC = \vec{AB}$

- Mark point c on $\vec{ab}$ and joint this to zero velocity point.

  Angular velocity of cylinder will be:

  $\omega_{ab} = V_{ab} = 5.61 \text{ rad/sec (c) AB}$

- Studying velocity of player will be

  $q_{b} = 4.1 \text{ m/s}$

  Absolute velocity of plunger $= 4.22 \text{ m/s q_c}$

- **Problem 9:** In a swiveling joint mechanism shown in figure link AB is the driving crank which rotates at 300 rpm clockwise. The length of the various links are:

  Determine,


- **Solution:**
Step 1: Draw the configuration diagram.

Step 2: Determine the velocity of point B with respect to A.

\[ V_b = \frac{2\times 300}{60} \times 0.1 = 3.14 \text{ m/s} \]

Step 3: Draw the velocity vector diagram choosing a suitable scale. o Mark zero velocity point a, d, o, g.

Velocity vector diagram

- From ‘a’ draw a line \( \rightarrow \) to AB and equal to 3.14 m/s.
- From ‘b’ draw a line \( \rightarrow \) to DC to intersect at C.
- Mark a point ‘e’ on vector bc such that be \( \rightarrow \) be
  \[ \rightarrow \rightarrow \text{ BC} \]
- From ‘e’ draw a line \( \overrightarrow{ef} \) to PE and from ‘a,d’ draw a line along PE to intersect at P.
- Extend the vector \( \overrightarrow{ep} \) to \( \overrightarrow{ef} \) such that \( \overrightarrow{ef} \parallel \overrightarrow{ef} \times \overrightarrow{EF} \)
- From ‘f’ draw a line \( \overrightarrow{ef} \) to Sf and from zero velocity point draw a line along the slider ‘S’ to intersect the previously drawn line at S.
- Velocity of slider \( \overrightarrow{gS} \parallel 2.6 \text{m/s} \). Angular Velocity of link EF.
- Velocity of link F in the swivel block = \( \overrightarrow{OP} \parallel 1.85 \text{ m/s} \).

**Problem 10:** Figure shows two wheels 2 and 4 which rolls on a fixed link 1. The angular uniform velocity of wheel is 2 is 10 rad/sec. Determine the angular velocity of links 3 and 4, and also the relative velocity of point D with respect to point E.

**Solution:**

**Step 1:** Draw the configuration diagram.

**Step 2:** Given \( \dot{\theta}_2 = 10 \text{ rad/sec} \). Calculate velocity of B with respect to G.

\[
\begin{align*}
V_b &= \dot{\theta}_2 \times BG \\
V_b &= 10 \times 43 = 430 \text{ mm/sec}.
\end{align*}
\]

**Step 3:** Draw the velocity vector diagram by choosing a suitable scale.
Redrawn configuration diagram

- Velocity vector diagram

- Problem 11: For the mechanism shown in figure link 2 rotates at constant angular velocity of 1 rad/sec construct the velocity polygon and determine.
  i) Velocity of point D.
  ii) Angular velocity of link BD.
  iii) Velocity of slider C.

- Solution:
Step 1: Draw configuration diagram.

Step 2:
Determine velocity of A with respect to O₂.

\[ V_b = \overrightarrow{O₂A} \]
\[ V_b = 1 \times 50.8 = 50.8 \text{ mm/sec.} \]

Step 3: Draw the velocity vector diagram, locate zero velocity points \( O₂O₆ \).

- From \( O₂ \), \( O₆ \) draw a line \( \square \) to \( O₂A \) in the direction of rotation equal to 50.8 mm/sec.
- From a draw a line \( \square \) to \( Ac \) and from \( O₂ \), \( O₆ \) draw a line along the line of stocks of c to intersect the previously drawn line at c.
- Mark point b on vector ac such that \( \overrightarrow{ab} \times \overrightarrow{AB} \) AC
- From b draw a line \( \square \) to BD and from \( O₂ \), \( O₆ \) draw a line \( \square \) to \( O₆D \) to intersect at d.

Step 4: \( V_d = O₆d = 32 \text{ mm/sec} \)

ADDITIONAL PROBLEMS FOR PRACTICE
• **Problem 1**: In a slider crank mechanism shown in offset by a perpendicular distance of 50 mm from the centre C. AB and BC are 750 mm and 200 mm long respectively crank BC is rotating at a uniform speed of 200 rpm. Draw the velocity vector diagram and determine velocity of slider A and angular velocity of link AB.

![Diagram of Problem 1](image1)

• **Problem 2**: For the mechanism shown in figure determine the velocities at points C, E and F and the angular velocities of links, BC, CDE and EF.

![Diagram of Problem 2](image2)

• The crank op of a crank and slotted lever mechanism shown in figure rotates at 100 rpm in the CCW direction. Various lengths of the links are OP = 90 mm, OA = 300 mm, AR = 480 mm and RS = 330 mm. The slider moves along an axis perpendicular to AO and in 120 mm from O. Determine the velocity of the slider when |AOP| is 135° and also mention the maximum velocity of slider.

![Diagram of Problem 3](image3)
• **Problem 4**: Find the velocity of link 4 of the scotch yoke mechanism shown in figure. The angular speed of link 2 is 200 rad/sec CCW, link O₂P = 40 mm.

\[\text{Q on link 4}\]

- **Problem 5**: In the mechanism shown in figure link AB rotates uniformly in C direction at 240 rpm. Determine the linear velocity of B and angular velocity of EF.

\[\begin{align*}
\text{AB} &= 160 \text{ mm} \\
\text{BC} &= 160 \text{ mm} \\
\text{CD} &= 100 \text{ mm} \\
\text{AD} &= 200 \text{ mm} \\
\text{EF} &= 200 \text{ mm} \\
\text{CE} &= 40 \text{ mm}
\end{align*}\]

**II Method**

- **Instantaneous Method**

To explain instantaneous centre let us consider a plane body P having a nonlinear motion relative to another body q consider two points A and B on body P having velocities as \(V_a\) and \(V_b\) respectively in the direction shown.
If a line is drawn to $V_a$, at A the body can be imagined to rotate about some point on the line. Thirdly, centre of rotation of the body also lies on a line to the direction of $V_b$ at B. If the intersection of the two lines is at I, the body P will be rotating about I at that instant. The point I is known as the instantaneous centre of rotation for the body P. The position of instantaneous centre changes with the motion of the body.

![Fig. 2](image1)

In case of the lines drawn from A and B meet outside the body P as shown in Fig 2.

![Fig. 3](image2)

If the direction of $V_a$ and $V_b$ are parallel to the at A and B met at . This is the case when the body has linear motion.

- **Number of Instantaneous Centers**

  The number of instantaneous centers in a mechanism depends upon number of links. If $N$ is the number of instantaneous centers and $n$ is the number of links.

- **Types of Instantaneous Centers**

  There are three types of instantaneous centers namely fixed, permanent and neither fixed nor permanent.

Example: Four bar mechanism. $n = 4$. 

\[
\begin{array}{cccc}
\text{4} & \text{4} & \text{1} \\
\end{array}
\]

\[
N = 6
\]
Fixed instantaneous center \( I_{12}, \ I_{14} \)
Permanent instantaneous center \( I_{23}, \ I_{34} \)
Neither fixed nor permanent instantaneous center \( I_{13}, \ I_{24} \)

- **Arnold Kennedy theorem of three centers:**

  **Statement:** If three bodies have motion relative to each other, their instantaneous centers should lie in a straight line.

  **Proof:**

  Consider a three link mechanism with link 1 being fixed link 2 rotating about \( I_{12} \) and link 3 rotating about \( I_{13} \). Hence, \( I_{12} \) and \( I_{13} \) are the instantaneous centers for link 2 and link 3. Let us assume that instantaneous center of link 2 and 3 be at point A i.e. \( I_{23} \). Point A is a coincident point on link 2 and link 3.

  Considering A on link 2, velocity of A with respect to \( I_{12} \) will be a vector \( \mathbf{V}_{A2} \) to link A \( I_{12} \). Similarly for point A on link 3, velocity of A with respect to \( I_{13} \) will be \( \mathbf{V}_{A3} \) to link A \( I_{13} \). It is seen that velocity vector of \( \mathbf{V}_{A2} \) and \( \mathbf{V}_{A3} \) are in different directions which is impossible. Hence, the instantaneous center of the two links cannot be at the assumed position.

  It can be seen that when \( I_{23} \) lies on the line joining \( I_{12} \) and \( I_{13} \) the \( \mathbf{V}_{A2} \) and \( \mathbf{V}_{A3} \) will be same in magnitude and direction. Hence, for the three links to be in relative motion all the three centers should lie in a same straight line. Hence, the proof.

  **Steps to locate instantaneous centers:**

  **Step 1:** Draw the configuration diagram.
  **Step 2:** Identify the number of instantaneous centers by using the relation \( n \) \( l \) \( n \)

  \[
  N = \frac{l}{2} 
  \]

  **Step 3:** Identify the instantaneous centers by circle diagram.
Step 4: Locate all the instantaneous centers by making use of Kennedy’s theorem.

To illustrate the procedure let us consider an example.

A slider crank mechanism has lengths of crank and connecting rod equal to 200 mm and 200 mm respectively locate all the instantaneous centers of the mechanism for the position of the crank when it has turned through 30° from IOC. Also find velocity of slider and angular velocity of connecting rod if crank rotates at 40 rad/sec.

Step 1: Draw configuration diagram to a suitable scale.

Step 2: Determine the number of links in the mechanism and find number of instantaneous centers. \( n = 4\) links \( N = \frac{4\times 4}{2} = 6 \)

Step 3: Identify instantaneous centers. o Suit it is a 4-bar link the resulting figure will be a square.

\[
\begin{align*}
1 & \quad 2 \\
I_{12} & \quad 2
\end{align*}
\]
Locate fixed and permanent instantaneous centers. To locate neither fixed nor permanent instantaneous centers use Kennedy’s three centers theorem.

**Step 4:** Velocity of different points. $V_a = 2 \times A_{12} = 40 \times 0.2 = 8 \text{ m/s}$ also $V_a = 2 \times A_{13}$

$3 = V_a A_{13}$

$V_b = 3 \times B_{13} = \text{Velocity of slider.}$
A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed. It reduces speed and increases torque. To create large gear ratio, gears are connected together to form gear trains. They often consist of multiple gears in the train.

The most common of the gear train is the gear pair connecting parallel shafts. The teeth of this type can be spur, helical or herringbone. The angular velocity is simply the reverse of the tooth ratio.

Any combination of gear wheels employed to transmit motion from one shaft to the other is called a gear train. The meshing of two gears may be idealized as two smooth discs with their edges touching and no slip between them. This ideal diameter is called the Pitch Circle Diameter (PCD) of the gear.

**Simple Gear Trains**

The typical spur gears as shown in diagram. The direction of rotation is reversed from one gear to another. It has no affect on the gear ratio. The teeth on the gears must all be the same size so if gear A advances one tooth, so does B and C.

\[ t = \text{number of teeth on the gear}, \]
\[ D = \text{Pitch circle diameter}, \quad N = \text{speed in rpm} \]
\[ m = \text{module} = \frac{D}{t} \]

and module must be the same for all gears otherwise they would not mesh.

\[ \frac{D_A}{D_B} \frac{D}{m} = \frac{t_C}{t_A} \]

\[ D_A = m t_A; \quad D_B = m t_B \quad \text{and} \quad D_C = m t_C \]

\[ \square = \text{angular velocity}. \ ]
\[ v = \text{linear velocity on the circle. } v = \frac{\theta}{t} = \frac{\theta}{2\pi} r \] (Idler gear)

**Application:**
a) to connect gears where a large center distance is required  
b) to obtain desired direction of motion of the driven gear (CW or CCW)  
c) to obtain high speed ratio

**Torque & Efficiency**

*The power transmitted by a torque \( T \ N\text{-m} \) applied to a shaft rotating at \( N \ \text{rev/min} \) is given by:*

\[ P = \frac{2}{60} NT \]

*In an ideal gear box, the input and output powers are the same so;*

\[ P = \frac{2}{60} N_1 T_1 = \frac{2}{60} N_2 T_2 \]

\[ T_2 = \frac{N_1}{N_2} \frac{1}{GR} \]

\[ N_1 T_1 \quad N_2 T_2 \]

\[ T_1 \quad N_2 \]

*It follows that if the speed is reduced, the torque is increased and vice versa. In a real gear box, power is lost through friction and the power output is smaller than the power input. The efficiency is defined as:*

\[ \text{Power out} = \frac{2}{60} N_2 T_2 \]

\[ \text{Power In} = \frac{2}{60} N_1 T_1 \]

*Because the torque in and out is different, a gear box has to be clamped in order to stop the case or body rotating. A holding torque \( T_3 \) must be applied to the body through the clamps. The total torque must add up to zero. *

\[ T_1 + T_2 + T_3 = 0 \]

*If we use a convention that anti-clockwise is positive and clockwise is negative we can determine the holding torque. The direction of rotation of the output shaft depends on the design of the gear box.*

**Compound Gear train**
Compound gears are simply a chain of simple gear trains with the input of the second being the output of the first. A chain of two pairs is shown below. Gear B is the output of the first pair and gear C is the input of the second pair. Gears B and C are locked to the same shaft and revolve at the same speed.

For large velocities ratios, compound gear train arrangement is preferred.

The velocity of each tooth on A and B are the same 

GEAR 'D' so: $\frac{A}{B} t_A = \frac{B}{B} t_B$ - as they are simple gears.

Likewise for C and D, $\frac{C}{D} t_C = \frac{D}{D} t_D$.

Reverted Gear train

The driver and driven axes lies on the same line. These are used in speed reducers, clocks and machine tools.

\[
GR = \begin{array}{c}
NA \\
ND \\
tA \\
tB \\
tC \\
tD
\end{array}
\]

If $R$ and $T$=Pitch circle radius & number of teeth of the gear

\[R_A + R_B = R_C + R_D \text{ and } t_A + t_B = t_C + t_D\]
**Epicyclic gear train:**

Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun. Here is a picture of a typical gear box.

This design can produce large gear ratios in a small space and are used on a wide range of applications from marine gearboxes to electric screwdrivers.

**Basic Theory**

The diagram shows a gear B on the end of an arm. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.

First consider what happens when the planet gear orbits the sun gear.

Observe point p and you will see that gear B also revolves once on its own axis. Any object orbiting around a center must rotate once. Now consider that B is free to rotate on its shaft and meshes with C.

Suppose the arm is held stationary and gear C is rotated once. B spins about its own center and the

Now consider that C is unable to rotate and the arm A is revolved once. Gear B will revolve
because of the orbit. It is this extra rotation that causes confusion. One way to get round this is to imagine that the whole system is revolved once. Then identify the gear that is fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up.

The following tabular method makes it easy.

Suppose gear $C$ is fixed and the arm $A$ makes one revolution. Determine how many revolutions the planet gear $B$ makes.

Step 1 is to revolve everything once about the center.

Step 2 identify that $C$ should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions of $B$.

Step 3 is simply add them up and we find the total revs of $C$ is zero and for the arm is 1.

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Revolve all once</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Revolve $C$ by $-1$ revolution, keeping the arm fixed</td>
<td>0</td>
<td>$\frac{t_c}{t_B}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>3</td>
<td>Add</td>
<td>1</td>
<td>$\frac{1}{t_c} \frac{t_c}{t_B}$</td>
<td>0</td>
</tr>
</tbody>
</table>

The number of revolutions made by $B$ is $\frac{t_c}{t_B}$. Note that if $C$ revolves $-1$, then the direction of $B$ is opposite.
Example: A simple epicyclic gear has a fixed sun gear with 100 teeth and a planet gear with 50 teeth. If the arm is revolved once, how many times does the planet gear revolve?

Solution:

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Revolve all once</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Revolve C by –1 revolution, keeping the arm fixed</td>
<td>0</td>
<td>100</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Add</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Gear B makes 3 revolutions for every one of the arm.

The design so far considered has no identifiable input and output. We need a design that puts an input and output shaft on the same axis. This can be done several ways.

Problem 1: In an epicyclic gear train shown in figure, the arm A is fixed to the shaft S. The wheel B having 100 teeth rotates freely on the shaft S. The wheel F having 150 teeth driven separately. If the arm rotates at 200 rpm and wheel F at 100 rpm in the same direction; find (a) number of teeth on the gear C and (b) speed of wheel B.

Solution:

\[ T_B = 100; \quad T_F = 150; \quad N_A = 200 \text{rpm}; \quad N_F = 100 \text{rpm}; \]
Since the module is same for all gears: the number of teeth on the gears is proportional to the pitch circle:

The gear B and gear F rotate in the opposite directions:

**The Gear B rotates at 350 rpm in the same direction of gears F and Arm A.**

**Problem 2:** In a compound epicyclic gear train as shown in the figure, has gears A and an annular gears D & E free to rotate on the axis P. B and C is a compound gear rotate about axis Q. Gear A rotates at 90 rpm CCW and gear D rotates at 450 rpm CW. Find the speed and direction of rotation of arm F and gear E. Gears A,B and C are having 18, 45 and 21 teeth respectively. All gears having same module and pitch.

![Diagram of compound epicyclic gear train]

**Solution:**

\[
T_A = 18; \quad T_B = 45; \quad T_C = 21; \quad N_A = -90 \text{ rpm}; \quad N_D = 10 \text{ rpm};
\]

Since the module and pitch are same for all gears:

the number of teeth on the gears is proportional to the pitch circle:

\[
r_D = r_A = r_B = r_C
\]

\[
T_D = T_A = T_B = T_C
\]

\[
T_D = 18 \div 45 \div 21 = 84 \text{ teeth on gear D}
\]

Gears A and D rotates in the opposite directions:

\[
TB \quad TD \quad NA \quad NF
\]
**Problem 3:** In an epicyclic gear of sun and planet type shown in figure 3, the pitch circle diameter of the annular wheel $A$ is to be nearly 216mm and module 4mm. When the annular ring is stationary, the spider that carries three planet wheels $P$ of equal size to make *one revolution* for every *five revolution* of the driving spindle carrying the sun wheel. Determine the number of teeth for all the wheels and the exact pitch circle diameter of the annular wheel. If an input torque of 20 N-m is applied to the spindle carrying the sun wheel, determine the fixed torque on the annular wheel.

**Solution:** Module being the same for all the meshing gears:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Spider arm L</th>
<th>Sun Wheel S</th>
<th>Planet wheel P</th>
<th>Annular wheel A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm L is fixed &amp; Sun wheel S is given +1 revolution</td>
<td>0</td>
<td>+1</td>
<td>$T_S T_P$</td>
<td>$T_A = 54$</td>
</tr>
<tr>
<td>Multiply by $m$ (S rotates through $m$ revolution)</td>
<td>0</td>
<td>$m$</td>
<td>$T_S m$</td>
<td>$T_A$</td>
</tr>
<tr>
<td>Add $n$ revolutions to all elements</td>
<td>$n$</td>
<td>$m+n$</td>
<td>$n T_S m$</td>
<td>$n T_A$</td>
</tr>
</tbody>
</table>

![Diagram](image-url)
Problem 4: The gear train shown in figure 4 is used in an indexing mechanism of a milling machine. The drive is from gear wheels A and B to the bevel gear wheel D through the gear train. The following table gives the number of teeth on each gear.

How many revolutions does D makes for one Figure 4 revolution of A under the following situations:

a. If A and B are having the same speed and same direction
b. If A and B are having the same speed and opposite direction
c. If A is making 72 rpm and B is at rest
d. If A is making 72 rpm and B 36 rpm in the same direction

Solution:

Gear D is external to the epicyclic train and thus C and D constitute an ordinary train.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Arm C (60)</th>
<th>E (28)</th>
<th>F (24)</th>
<th>A (72)</th>
<th>B (72)</th>
<th>G (28)</th>
<th>H (24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm or C is fixed &amp; wheel A is</td>
<td>0</td>
<td>-1</td>
<td>24</td>
<td>6</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>given +1 revolution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>28 7</td>
</tr>
<tr>
<td>Multiply by m</td>
<td>0</td>
<td>-m</td>
<td>7</td>
<td>m</td>
<td>-m</td>
<td>m</td>
<td>7</td>
</tr>
<tr>
<td>(A rotates through m revolution)</td>
<td></td>
<td></td>
<td>6m</td>
<td></td>
<td></td>
<td></td>
<td>6m</td>
</tr>
<tr>
<td>Add n revolutions to all elements</td>
<td>n</td>
<td>n-m</td>
<td>7</td>
<td>n+m</td>
<td>n-m</td>
<td>n+m</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>n 6m</td>
<td></td>
<td></td>
<td></td>
<td>n 6m</td>
</tr>
</tbody>
</table>

(i) For one revolution of A: \( n + m = 1 \) (1)
For A and B for same speed and direction: \[ n + m = n - m \] (2)

From (1) and (2): \( n = 1 \) and \( m = 0 \)

If C or arm makes one revolution, then revolution made by D is given by:

\[
\begin{align*}
ND & \quad TC \quad 60 \\
NC & \quad TD \quad 30 \\
\hline
& \quad 2
\end{align*}
\]

(i) A and B same speed, opposite direction:

\[
(n + m) = -(n - m) \quad (3) \quad n = 0; \quad m = 1
\]

When C is fixed and A makes one revolution, D does not make any revolution.

(iii) A is making 72 rpm: \( (n + m) = 72 \)

B at rest \( (n - m) = 0 \) \( n = m = 36 \) rpm

C makes 36 rpm and D makes 36\( \square 30 \square 72 \) rpm

(iv) A is making 72 rpm and B making 36 rpm

\[
(n + m) = 72 \text{ rpm} \quad \text{and} \quad (n - m) = 36 \text{ rpm}
\]

\[
(n + (n - m)) = 72; \quad n = 54
\]

D makes 54\( \square 30 \square 108 \) rpm

Problem 5: Figure 5 shows a compound epicyclic gear train, gears \( S_1 \) and \( S_2 \) being rigidly attached to the shaft \( Q \). If the shaft \( P \) rotates at 1000 rpm clockwise, while the annular \( A_2 \) is driven in counter clockwise direction at 500 rpm, determine the speed and direction of rotation of shaft \( Q \). The number of teeth in the wheels are \( S_1 = 24; \) \( S_2 = 40; \) \( A_1 = 100; \) \( A_2 = 120. \)

Solution: Consider the gear train \( P A_1 S_1 \):
<table>
<thead>
<tr>
<th>Operation</th>
<th>Arm P</th>
<th>( A_1 ) (100)</th>
<th>( S_1(24) )</th>
<th>OR</th>
<th>Operation</th>
<th>Arm P</th>
<th>( A_1 ) (100)</th>
<th>( S_1(24) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm ( P ) is fixed &amp; wheel ( A_1 ) is given +1 revolution</td>
<td>0</td>
<td>+1</td>
<td>( \overline{100} ) ( P_1 )</td>
<td>24</td>
<td>Arm ( P ) is fixed &amp; wheel ( A_1 ) is given -1 revolution</td>
<td>0</td>
<td>-1</td>
<td>( \overline{100} ) ( P_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 25 ) ( \overline{6} )</td>
<td></td>
<td></td>
<td>0</td>
<td>-1</td>
<td>( \overline{6} ) ( \overline{1} ) ( \overline{6} )</td>
</tr>
<tr>
<td>Multiply by ( m )</td>
<td>0</td>
<td>+m</td>
<td>( \overline{25} ) ( \overline{6} ) m</td>
<td></td>
<td>Add +1 revolutions to all elements</td>
<td>+1</td>
<td>0</td>
<td>( \overline{25} ) ( \overline{31} ) ( \overline{6} )</td>
</tr>
<tr>
<td>( (A_1 ) rotates through ( m ) revolution)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Add +1 revolutions to all elements</td>
<td>+1</td>
<td>0</td>
<td>( \overline{25} ) ( \overline{31} ) ( \overline{6} )</td>
</tr>
</tbody>
</table>

If \( A_1 \) is fixed: \( n + m \) gives \( n = -m \)

Now consider whole gear train:

<table>
<thead>
<tr>
<th>Operation</th>
<th>( A_1 ) (100)</th>
<th>( A_2 ) (120)</th>
<th>( S_1(24), S_2(40) ) and ( Q )</th>
<th>Arm P</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 ) is fixed &amp; wheel ( A_2 ) is given +1 revolution</td>
<td>0</td>
<td>+1</td>
<td>( \overline{120} ) ( P_2 )</td>
<td>( \overline{6} ) ( \overline{3} ) ( \overline{31} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( P_2 ) ( 40 )</td>
<td>18 ( \overline{31} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \overline{3} )</td>
<td></td>
</tr>
<tr>
<td>Multiply by ( m )</td>
<td>0</td>
<td>+m</td>
<td>( \overline{3} ) m</td>
<td>( \overline{18} ) ( \overline{3} ) ( \overline{1} ) ( \overline{m} )</td>
</tr>
<tr>
<td>( (A_1 ) rotates through ( m ) revolution)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Add ( n ) revolutions to all elements</td>
<td>( n )</td>
<td>( n + m )</td>
<td>( n ) ( \overline{3} ) ( m )</td>
<td>( \overline{18} ) ( n ) ( \overline{3} ) ( \overline{1} ) ( \overline{m} )</td>
</tr>
</tbody>
</table>

When \( P \) makes 1000 rpm: \( n \) \( \overline{3} \) \( 1 \) \( m \) = 1000 (1)

\( A_2 \) makes – 500 rpm: \( n + m = -500 \) (2)

\( \overline{18} \) \( \overline{3} \) \( \overline{1} \) \( \overline{m} \) \( \overline{0} \) \( \overline{0} \) \( \overline{0} \)

from (1) and (2):

\( \overline{18} \) \( \overline{5} \) \( \overline{0} \) \( \overline{0} \) \( m \) \( \overline{3} \) \( \overline{1} \) \( m \) \( \overline{1} \) \( \overline{0} \) \( \overline{0} \)
and \( n = 949 \text{ rpm} \)

\( N_Q = n - 3 \, m = 449 - (3 \times -949) = 3296 \text{ rpm} \)
Chapter 6- SPUR GEARS

Spur gears: Spur gears are the most common type of gears. They have straight teeth, and are mounted on parallel shafts. Sometimes, many spur gears are used at once to create very large gear reductions. Each time a gear tooth engages a tooth on the other gear, the teeth collide, and this impact makes a noise. It also increases the stress on the gear teeth. To reduce the noise and stress in the gears, most of the gears in your car are helical.

Spur gears (Emerson Power Transmission Corp)

Spur gears are the most commonly used gear type. They are characterized by teeth, which are perpendicular to the face of the gear. Spur gears are most commonly available, and are generally the least expensive.

- **Limitations**: Spur gears generally cannot be used when a direction change between the two shafts is required.
- **Advantages**: Spur gears are easy to find, inexpensive, and efficient.

2. Parallel helical gears: The teeth on helical gears are cut at an angle to the face of the gear. When two teeth on a helical gear system engage, the contact starts at one end of the tooth and gradually spreads as the gears rotate, until the two teeth are in full engagement.

Helical gears
Herringbone gears

This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears. For this reason, helical gears are used in almost all car transmission.

Because of the angle of the teeth on helical gears, they create a thrust load on the gear when they mesh. Devices that use helical gears have bearings that can support this thrust load.

One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees.

Helical gears to have the following differences from spur gears of the same size:

- Tooth strength is greater because the teeth are longer,
- Greater surface contact on the teeth allows a helical gear to carry more load than a spur gear
- The longer surface of contact reduces the efficiency of a helical gear relative to a spur gear

Rack and pinion (The rack is like a gear whose axis is at infinity.): **Racks** are straight gears that are used to convert rotational motion to translational motion by means of a gear mesh. (They are in theory a gear with an infinite pitch diameter). In theory, the torque and angular velocity of the pinion gear are related to the Force and the velocity of the rack by the radius of the pinion gear, as is shown.

Perhaps the most well-known application of a rack is the rack and pinion steering system used on many cars in the past

**Gears for connecting intersecting shafts:** Bevel gears are useful when the direction of a shaft's rotation needs to be changed. They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well.

The teeth on bevel gears can be straight, spiral or hypoid. Straight bevel gear teeth actually have the same problem as straight spur gear teeth, as each tooth engages; it impacts the corresponding tooth all at once.

Just like with spur gears, the solution to this problem is to curve the gear teeth. These spiral teeth engage just like helical teeth: the contact starts at one end of the gear and progressively spreads across the whole tooth.
On straight and spiral bevel gears, the shafts must be perpendicular to each other, but they must also be in the same plane. The hypoid gear, can engage with the axes in different planes.

This feature is used in many car differentials. The ring gear of the differential and the input pinion gear are both hypoid. This allows the input pinion to be mounted lower than the axis of the ring gear. Figure shows the input pinion engaging the ring gear of the differential. Since the driveshaft of the car is connected to the input pinion, this also lowers the driveshaft. This means that the driveshaft doesn't pass into the passenger compartment of the car as much, making more room for people and cargo.

**Neither parallel nor intersecting shafts:** Helical gears may be used to mesh two shafts that are not parallel, although they are still primarily used in parallel shaft applications. A special application in which helical gears are used is a crossed gear mesh, in which the two shafts are perpendicular to each other.
Crossed-helical gears

Worm and worm gear: Worm gears are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater.

Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm. This is because the angle on the worm is so shallow that when the gear tries to spin it, the friction between the gear and the worm holds the worm in place.

This feature is useful for machines such as conveyor systems, in which the locking feature can act as a brake for the conveyor when the motor is not turning. One other very interesting usage of worm gears is in the Torsen differential, which is used on some high-performance cars and trucks.

4.3 Terminology for Spur Gears
Addendum: The radial distance between the Pitch Circle and the top of the teeth.

Arc of Action: Is the arc of the Pitch Circle between the beginning and the end of the engagement of a given pair of teeth.

Arc of Approach: Is the arc of the Pitch Circle between the first point of contact of the gear teeth and the Pitch Point.

Arc of Recession: That arc of the Pitch Circle between the Pitch Point and the last point of contact of the gear teeth.

Backlash: Play between mating teeth.

Base Circle: The circle from which is generated the involute curve upon which the tooth profile is based.

Center Distance: The distance between centers of two gears.

Chordal Addendum: The distance between a chord, passing through the points where the Pitch Circle crosses the tooth profile, and the tooth top.

Chordal Thickness: The thickness of the tooth measured along a chord passing through the points where the Pitch Circle crosses the tooth profile.

Circular Pitch: Millimeter of Pitch Circle circumference per tooth.

Circular Thickness: The thickness of the tooth measured along an arc following the Pitch Circle

Clearance: The distance between the top of a tooth and the bottom of the space into which it fits on the meshing gear.

Contact Ratio: The ratio of the length of the Arc of Action to the Circular Pitch.

Dedendum: The radial distance between the bottom of the tooth to pitch circle.

Diametral Pitch: Teeth per mm of diameter.

Face: The working surface of a gear tooth, located between the pitch diameter and the top of the tooth.

Face Width: The width of the tooth measured parallel to the gear axis.

Flank: The working surface of a gear tooth, located between the pitch diameter and the bottom of the teeth

Gear: The larger of two meshed gears. If both gears are the same size, they are both called "gears".

Land: The top surface of the tooth.
**Line of Action:** That line along which the point of contact between gear teeth travels, between the first point of contact and the last.

**Module:** Millimeter of Pitch Diameter to Teeth.

**Pinion:** The smaller of two meshed gears.

**Pitch Circle:** The circle, the radius of which is equal to the distance from the center of the gear to the pitch point.

**Diametral pitch:** Teeth per millimeter of pitch diameter.

**Pitch Point:** The point of tangency of the pitch circles of two meshing gears, where the Line of Centers crosses the pitch circles.

**Pressure Angle:** Angle between the Line of Action and a line perpendicular to the Line of Centers.

**Profile Shift:** An increase in the Outer Diameter and Root Diameter of a gear, introduced to lower the practical tooth number or achieve a non-standard Center Distance.

**Ratio:** Ratio of the numbers of teeth on mating gears.

**Root Circle:** The circle that passes through the bottom of the tooth spaces.

**Root Diameter:** The diameter of the Root Circle.

**Working Depth:** The depth to which a tooth extends into the space between teeth on the mating gear.

### 4.2 Gear-Tooth Action

#### 4.2.1 Fundamental Law of Gear-Tooth Action

Figure 5.2 shows two mating gear teeth, in which

- Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point \( K \).
- \( N_1N_2 \) is the common normal of the two profiles.
- \( N_i \) is the foot of the perpendicular from \( O_i \) to \( N_jN_2 \).
- \( N_2 \) is the foot of the perpendicular from \( O_2 \) to \( N_jN_2 \).
Although the two profiles have different velocities $V_1$ and $V_2$ at point $K$, their velocities along $N_1N_2$ are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other.

4.2.2 Constant Velocity Ratio

For a constant velocity ratio, the position of $P$ should remain unchanged. In this case, the motion transmission between two gears is equivalent to the motion transmission between two imagined slip-less cylinders with radius $R_1$ and $R_2$ or diameter $D_1$ and $D_2$. We can get two circles whose centers are at $O_1$ and $O_2$, and through pitch point $P$. These two circles are termed pitch circles. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

The fundamental law of gear-tooth action may now also be stated as follow (for gears with fixed center distance):

A common normal (the line of action) to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point.

Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves, and the relative rotation speed of the gears will be constant (constant velocity ratio).

4.2.3 Conjugate Profiles

To obtain the expected velocity ratio of two tooth profiles, the normal line of their profiles must pass through the corresponding pitch point, which is decided by the velocity ratio. The two profiles which satisfy this requirement are called conjugate profiles. Sometimes, we simply termed the tooth profiles which satisfy the fundamental law of gear-tooth action the conjugate profiles.

Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the cycloidal and involute profiles. The involute has important advantages; it is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required when using the involute profile. The most commonly used conjugate tooth curve is the involute curve. (Erdman & Sandor).

conjugate action: It is essential for correctly meshing gears, the size of the teeth (the module) must be the same for both the gears.

Another requirement - the shape of teeth necessary for the speed ratio to remain constant during an increment of rotation; this behavior of the contacting surfaces (ie. the teeth flanks) is known as conjugate action.

4.3 Involute Curve

The following examples are involute spur gears. We use the word involute because the contour of gear teeth curves inward. Gears have many terminologies, parameters and principles. One of the important concepts is the velocity ratio, which is the ratio of the rotary velocity of the driver gear to that of the driven gears.
4.1 Generation of the Involute Curve
The curve most commonly used for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string, which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the **base circle**.

![Involute curve](image)

**Figure 4.3 Involute curve**

4.2 Properties of Involute Curves
1. The line rolls without slipping on the circle.
2. For any instant, the *instantaneous center* of the motion of the line is its point of tangent with the circle.

Note: We have not defined the term *instantaneous center* previously. The **instantaneous center** or **instant center** is defined in two ways.

1. When two bodies have planar relative motion, the instant center is a point on one body about which the other rotates at the instant considered.
2. When two bodies have planar relative motion, the instant center is the point at which the bodies are relatively at rest at the instant considered.
3. The normal at any point of an involute is tangent to the base circle. Because of the property (2) of the involute curve, the motion of the point that is tracing the involute is perpendicular to the line at any instant, and hence the curve traced will also be perpendicular to the line at any instant.

There is no involute curve within the base circle.

**Cycloidal profile:**

![Cycloidal profile](image)
Epicycliodal Profile:

Hypocycliodal Profile:

The involute profile of gears has important advantages;

- It is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required. The most commonly used conjugate tooth curve is the involute curve. (Erdman & Sandor).

2. In involute gears, the pressure angle, remains constant between the point of tooth engagement and disengagement. It is necessary for smooth running and less wear of gears.

   But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts increasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.

3. The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively. Thus the involute teeth are easy to manufacture than cycloidal teeth.

   In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

Advantages of Cycloidal gear teeth:

1. Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.
2. In cycloidal gears, the contact takes place between a convex flank and a concave surface, where as in involute gears the convex surfaces are in contact. This condition results in less wear in cycloidal gears as compared to involute gears. However the difference in wear is negligible.

3. In cycloidal gears, the interference does not occur at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

**Properties of involute teeth:**

1. A normal drawn to an involute at pitch point is a tangent to the base circle.
2. Pressure angle remains constant during the mesh of an involute gears.
3. The involute tooth form of gears is insensitive to the centre distance and depends only on the dimensions of the base circle.
4. The radius of curvature of an involute is equal to the length of tangent to the base circle.
5. Basic rack for involute tooth profile has straight line form.
6. The common tangent drawn from the pitch point to the base circle of the two involutes is the line of action and also the path of contact of the involutes.
7. When two involutes gears are in mesh and rotating, they exhibit constant angular velocity ratio and is inversely proportional to the size of base circles. (Law of Gearing or conjugate action)
8. Manufacturing of gears is easy due to single curvature of profile.

**System of Gear Teeth**

The following four systems of gear teeth are commonly used in practice:

1. 14½° Composite system
2. 14½° Full depth involute system
3. 20° Full depth involute system
4. 20° Stub involute system

The 14½° composite system is used for general purpose gears. It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion.

The teeth are produced by formed milling cutters or hobs.

The tooth profile of the 14½° full depth involute system was developed using gear hobs for spur and helical gears.

The tooth profile of the 20° full depth involute system may be cut by hobs.

The increase of the pressure angle from 14½° to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base.

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**Involutometry**

The study of the geometry of the involute profile for gear teeth is called involutometry. Consider an involute of base circle radius \( r_a \) and two points B and C on the involute as shown in figure. Draw normal to the involute from the points B and C. The normal BE and CF are tangents to the Base circle.

![Involute Profile Diagram](image)
Let \( r_a \) = base circle radius of gear \( r_b \) = radius of point B on the involute \( r_c \) = radius of point C on the involute and

From the properties of the Involute:

\[ Arc \ AE = Length \ BE \ and \]

\[ Arc \ AF = Length \ CF \]

Using this equation and knowing tooth thickness at any point on the tooth, it is possible to calculate the thickness of the tooth at any point

**Number of Pairs of Teeth in Contact**

Continuous motion transfer requires two pairs of teeth in contact at the ends of the path of contact, though there is only one pair in contact in the middle of the path, as in Figure.

The average number of teeth in contact is an important parameter - if it is too low due to the use of inappropriate profile shifts or to an excessive centre distance. The manufacturing inaccuracies may lead to loss of kinematic continuity - that is to impact, vibration and noise.

The average number of teeth in contact is also a guide to load sharing between teeth; it is termed the contact ratio

**Length of path of contact for Rack and Pinion:**
Let

\[ r = \text{Pitch circle radius of the pinion} = O_1P \]
\[ \rho = \text{Pressure angle} \quad r_\rho = \text{Addendum radius of the pinion} \]
\[ a = \text{Addendum of the rack} \]

\[ EF = \text{Length of path of contact} \]

\[ EF = \text{Path of approach } EP + \text{Path of recess } PF \]

\[ P : \text{Path of approach } EP = a \]

\[ NP = O_1P \sin \theta + r \sin \theta \]

\[ O_1N = O_1P \cos \theta \]

From triangle \( O_1NF: \)

\[ NF = O_1F_2 + O_1N_2 - 12 \]

Substituting \( NP \) and \( NF \) values in the equation (3)
Path of races \[ PF \] \[ \frac{r^2}{r^2} \cos^2 \frac{r}{1^2} \sin \]

Exercise problems refer presentation slides

Interference in Involute Gears

Figure shows a pinion and a gear in mesh with their center as \( O_1 \) and \( O_2 \) respectively. \( MN \) is the common tangent to the basic circles and \( KL \) is the path of contact between the two mating teeth. Consider, the radius of the addendum circle of pinion is increased to \( O_1N \), the point of contact \( L \) will moves from \( L \) to \( N \). If this radius is further increased, the point of contact \( L \) will be inside of base circle of wheel and not on the involute profile of the pinion.

The tooth tip of the pinion will then undercut tooth on the wheel at the root and damages of the involute profile. This effect is known as interference, and occurs when the teeth are being cut and weakens the tooth at its root. In general, the phenomenon, when the tip of tooth undercuts the root on its mating gear is known as interference.
Similarly, if the radius of the addendum circles of the wheel increases beyond O₂M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called interference points.

Interference may be avoided if the path of the contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O₁N and of the wheel is O₂M.

The interference may only be prevented, if the point of contact between the two teeth is always on the involute profiles and if the addendum circles of the two mating gears cut the common tangent to the base circles at the points of tangency.

When interference is just prevented, the maximum length of path of contact is MN.

**Methods to avoid Interference**

1. Height of the teeth may be reduced.
2. Under cut of the radial flank of the pinion.
3. Centre distance may be increased. It leads to increase in pressure angle.
4. By tooth correction, the pressure angle, centre distance and base circles remain unchanged, but tooth thickness of gear will be greater than the pinion tooth thickness. Minimum number of teeth on the pinion avoid Interference

The pinion turns clockwise and drives the gear as shown in Figure.

Points M and N are called interference points. i.e., if the contact takes place beyond M and N, interference will occur.

The limiting value of addendum circle radius of pinion is O₁N and the limiting value of addendum circle radius of gear is O₂M. Considering the critical addendum circle radius of gear, the limiting number of teeth on gear can be calculated.

Let

\[ \Phi = \text{pressure angle} \]
\[ R = \text{pitch circle radius of gear} = \tfrac{1}{2} m T r = \]
\[ \text{pitch circle radius of pinion} = \tfrac{1}{2} m t T & t = \]
\[ \text{number of teeth on gear} \& \text{ pinion} \]
\[ m = \text{module} \]
\[ a_w = \text{Addendum constant of gear (or) wheel} \]
\[ a_p = \text{Addendum constant of pinion} \]
\[ a_w, m = \text{Addendum of gear} \]
\[ a_p, m = \text{Addendum of pinion} \]
Minimum number of teeth on the wheel avoid Interference

From triangle O2MP, applying cosine rule and simplifying, the limiting radius of wheel addendum circle:

Minimum number of teeth on the pinion for involute rack to avoid interference

The rack is part of toothe wheel of infinite diameter. The base circle diameter and profile of the involute teeth are straight lines.

Let $t =$ Minimum number of teeth on the pinion

Pitch circle radius of the pinion $= \frac{1}{2} mt$

$\alpha =$ Pressure angle

$A_{R,m} =$ Addendum of rack

The straight profiles of the rack are tangential to the pinion profiles at the point of contact and perpendicular to the tangent $PM$. Point L is the limit of interference.

**Backlash:**

The gap between the non-drive face of the pinion tooth and the adjacent wheel tooth is known as backlash.
If the rotational sense of the pinion were to reverse, then a period of unrestrained pinion motion would take place until the backlash gap closed and contact with the wheel tooth re-established impulsively.

Backlash is the error in motion that occurs when gears change direction. The term "backlash" can also be used to refer to the size of the gap, not just the phenomenon it causes; thus, one could speak of a pair of gears as having, for example, "0.1 mm of backlash."

A pair of gears could be designed to have zero backlash, but this would presuppose perfection in manufacturing, uniform thermal expansion characteristics throughout the system, and no lubricant.

Therefore, gear pairs are designed to have some backlash. It is usually provided by reducing the tooth thickness of each gear by half the desired gap distance.

In the case of a large gear and a small pinion, however, the backlash is usually taken entirely off the gear and the pinion is given full sized teeth.

Backlash can also be provided by moving the gears farther apart. For situations, such as instrumentation and control, where precision is important, backlash can be minimised through one of several techniques.

Let \( r \) = standard pitch circle radius of pinion

\( R \) = standard pitch circle radius of wheel

\( c \) = standard centre distance = \( r + R \)

\( r' \) = operating pitch circle radius of pinion
\( R' \) = operating pitch circle radius of wheel
\( c' = \) operating centre distance = \( r' + R' \)
\( \Phi = \) Standard pressure angle
\( \Phi' = \) operating pressure angle
\( h = \) tooth thickness of pinion on standard pitch circle = \( p/2 \)
\( h' = \) tooth thickness of pinion on operating pitch circle

Let
\[
H = \text{tooth thickness of gear on standard pitch circle}
\]
\[
H_1 = \text{tooth thickness of gear on operating pitch circle}
\]
\[
p = \text{standard circular pitch} = 2\pi \frac{r}{t} = 2\pi \frac{R}{T}
\]
\[
p' = \text{operating circular pitch} = 2\pi \frac{r_1}{t} = 2\pi \frac{R_1}{T}
\]
\[
\Delta C = \text{change in centre distance} \ B
\]
\[
= \text{Backlash} \quad t = \text{number of teeth on pinion}
\]
\[
T = \text{number of teeth on gear.}
\]

Involute gears have the invaluable ability of providing conjugate action when the gears’ centre distance is varied either deliberately or involuntarily due to manufacturing and/or mounting errors.

There is an infinite number of possible centre distances for a given pair of profile shifted gears, however we consider only the particular case known as the extended centre distance.

**Non Standard Gears:**
The important reason for using non standard gears are to eliminate undercutting, to prevent interference and to maintain a reasonable contact ratio.

The two main non-standard gear systems:
1. Long and short Addendum system and
2. Extended centre distance system.

Long and Short Addendum System:
The addendum of the wheel and the addendum of the pinion are generally made of equal lengths.

Here the profile/rack cutter is advanced to a certain increment towards the gear blank and the same quantity of increment will be withdrawn from the pinion blank.
Therefore an increased addendum for the pinion and a decreased addendum for the gear is obtained. The amount of increase in the addendum of the pinion should be exactly equal to the addendum of the wheel is reduced.

The effect is to move the contact region from the pinion centre towards the gear centre, thus reducing approach length and increasing the recess length. In this method there is no change in pressure angle and the centre distance remains standard.

**Extended centre distance system:**

Reduction in interference with constant contact ratio can be obtained by increasing the centre distance. The effect of changing the centre distance is simply in increasing the pressure angle.

In this method when the pinion is being cut, the profile cutter is withdrawn a certain amount from the centre of the pinion so the addendum line of the cutter passes through the interference point of pinion. The result is increase in tooth thickness and decrease in tooth space.

Now If the pinion is meshed with the gear, it will be found that the centre distance has been increased because of the decreased tooth space. Increased centre distance will have two undesirable effects.