UNIT 1

DIMENSIONAL ANALYSIS AND MODEL STUDIES

Introduction

Dimensional analysis is a mathematical technique which makes use of the study of dimensions as an aid to the solution of several engineering problems. It deals with the dimensions of the physical quantities are measured by comparison, which is made with respect to an arbitrarily fixed value. Length L, mass M and Time T are three fixed dimensions which are of importance in fluid mechanics. If in any problem of fluid mechanics, heat is involved then temperature is also taken as fixed dimension. These fixed dimensions are called fundamental dimensions or fundamental quantity.

Secondary units or Derived units

Secondary or Derived quantities are those quantities which possess more than one fundamental dimension. For example velocity is defined by distance per unit time (L/T), density by mass per unit volume (M/L^3) and acceleration by distance per second square (L/T^2). Then the velocity, density and acceleration become as secondary or derived quantities. The expressions (L/T), (M/L^3) and (L/T^2) are called the dimensions of velocity, density and acceleration respectively. The dimensions of mostly used physical quantities in fluid mechanics are given in table 1.

Table 1. Dimensions of Physical quantities

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Physical Quantity</th>
<th>Symbol</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) Fundamental</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Length</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>3</td>
<td>Mass</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>4</td>
<td>Time</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>5</td>
<td>(b) Geometric</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Area</td>
<td>A</td>
<td>L^2</td>
</tr>
<tr>
<td>7</td>
<td>Volume</td>
<td>V</td>
<td>L^3</td>
</tr>
<tr>
<td>8</td>
<td>(c) Kinematic quantities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Velocity</td>
<td>v</td>
<td>LT^{-1}</td>
</tr>
<tr>
<td>10</td>
<td>Angular velocity</td>
<td>ω</td>
<td>T^{-1}</td>
</tr>
<tr>
<td>11</td>
<td>Acceleration</td>
<td>A</td>
<td>LT^{-2}</td>
</tr>
<tr>
<td>12</td>
<td>Angular acceleration</td>
<td>A</td>
<td>T^2</td>
</tr>
</tbody>
</table>
**Dimensional homogeneity**

Dimensional homogeneity means the dimensions of each terms in an equation on both sides are equal. Thus if the dimensions of each term on both sides of an equation are the same the equation is known as dimensionally homogenous equation. The powers of fundamental dimensions i.e., L, M, T on both sides of the equation will be identical for a dimensionally homogenous equation. Such equations are independent of the system of units.

Let us consider the equation $V = u + at$

Dimensions of L.H.S = $V = L/T = LT^{-1}$

Dimensions of R.H.S = $LT^{-1} + (LT^{-2}) (T)$

$$= LT^{-1} + LT^{-1}$$

$$= LT^{-1}$$

Dimensions of L.H.S = Dimensions of R.H.S = $LT^{-1}$

Therefore, equation $V = u + at$ is dimensionally homogeneous.
Uses of Dimensional Analysis

1. It is used to test the dimensional homogeneity of any derived equation.
2. It is used to derive equation.
3. Dimensional analysis helps in planning model tests.

Methods of Dimensional Analysis

If the number of variable involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods.

1. Rayleigh’s method
2. Buckingham’s (π – theorem) method

1. Rayleigh’s method

Rayleigh’s method of analysis is adopted when number of parameters or variables is less (3 or 4 or 5). If the number of independent variables becomes more than four, then it is very difficult to find the expression for the dependent variable.

Methodology

Let X be a variable, which depends on $X_1, X_2$ and $X_3$ variables. Then according to this method X is function of $X_1, X_2$ and $X_3$ and mathematically written as

$$X = f(X_1, X_2, X_3).$$

This can also be written as $X = K X_1^a X_2^b X_3^c$ where k is constant and a, b, c are arbitrary powers.

Dimensions for quantities on left hand side as well as on the right hand side are written and using the concept of Dimensional Homogeneity a, b and c can be determined.

Problem 1: Velocity of sound in air varies as bulk modulus of electricity K, Mass density $\rho$. Derive an expression for velocity in form $C = \sqrt{\frac{K}{\rho}}$
Solution: \( C = f (K, \rho) \)

\[ C = M K^a \rho^b \]

M – Constant of proportionality

\[ [C] = [K]^a \cdot [\rho]^b \]

\[ [LM_0 T^{-1}] = [L^{-1} M T^{-2}]^a [L^2 M T_0]^b \]

\[ -a - 3b = 1 \]

\[ a + b = 0 \]

\[ -2b = 1 \]

\[ b = -\frac{1}{2} \]

\[ a = \frac{1}{2} \]

\[ C = MK^{1/2} \rho^{1/2} \]

\[ C = M \sqrt{\frac{K}{\rho}} \]

If \( M = 1 \) \( C = \sqrt{\frac{K}{\rho}} \)

Problem 2: Find the equation for the power developed by a pump if it depends on head \( H \) discharge \( Q \) and specific weight \( \gamma \) of the fluid.

Solution: \( P = f (H, Q, \gamma) \)

\[ P = K \cdot H^a \cdot Q^b \cdot \gamma^c \]

\[ [P] = [H]^a \cdot [Q]^b \cdot [\gamma]^c \]

\[ [L^2 M T^{-3}] = [LM_0 T^0]^a \cdot [L^2 M T^{-2}]^b \cdot [L^{-2} M T^{-2}]^c \]

\[ 2 = a + 3b - 2c \]

\[ 1 = c \]

\[ -3 = -b - 2 \]

\[ b = -2 + 3 \]

\[ b = 1 \]

\[ 2 = a + 3 - 2 \]

\[ a = 1 \]

\[ P = K \cdot H^1 \cdot Q^1 \cdot \gamma^1 \]

\[ \text{Power} = L^2 M T^{-3} \]

\[ \text{Head} = LM_0 T^0 \]

\[ \text{Discharge} = L^3 M_0 T^{-1} \]

\[ \text{Specific Weight} = L^{-2} M T^{-2} \]
\[ P = K \cdot H \cdot Q \cdot \gamma \]

When, \( K = 1 \)

\[ P = H \cdot Q \cdot \gamma \]

**Problem 3:** Find an expression for drag force \( R \) on a smooth sphere of diameter \( D \) moving with uniform velocity \( V \) in a fluid of density \( \rho \) and dynamic viscosity \( \mu \).

**Solution:**

\[ R = f(D, V, \rho, \mu) \]

\[ R = K \cdot D^a \cdot V^b \cdot \rho^c \cdot \mu^d \]

\[ [R] = [D]^a \cdot [V]^b \cdot [\rho]^c \cdot [\mu]^d \]

\[ [\text{LMT}^{-2}] = [\text{LMT}^0]^a \cdot [\text{L}^{-3}\text{MT}^{-1}]^b \cdot [\text{L}^{-1}\text{MT}^{-1}]^c \cdot [\text{L}^{-1}\text{MT}^{-1}]^d \]

\[ c + d = 1 \]

\[ c = 1 - d \]

\[ -b - d = -2 \]

\[ b = 2 - d \]

\[ 1 = a + b - 3c - d \]

\[ 1 = a + 2 - d - 3(1 - d) - d \]

\[ 1 = a + 2 - d - 3 + 3d - d \]

\[ a = 2 - d \]

\[ R = K \cdot D^{2-d} \cdot V^{2-d} \cdot \rho^{1-d} \cdot \mu^d \]

\[ R = K \cdot (D^2/D) \cdot (V^2/V^d) \cdot (\rho/\rho^d) \cdot \mu^d \]

\[ R = K \cdot \rho \cdot V^2 \cdot D^2 \left[ \mu/\rho V D \right]^d \]

\[ R = \rho \cdot V^2 \cdot D^2 \Phi[\mu/\rho V D] \]

\[ R = \rho \cdot V^2 \cdot D^2 \Phi[\rho V D/\mu] \]

\[ R = \rho V^2 D^2 \Phi[N R_e] \]
2. Buckingham’s \( \Pi \) Method

If there are \( n \) – variables in a physical phenomenon and those \( n \)-variables contain ‘\( m \)’ dimensions, then the variables can be arranged into \( (n-m) \) dimensionless groups called \( \Pi \) terms. If \( f (X_1, X_2, X_3, \ldots, X_n) = 0 \) and variables can be expressed using \( m \) dimensions then:

\[
f (\Pi_1, \Pi_2, \Pi_3, \ldots, \Pi_{n-m}) = 0
\]

Where, \( \Pi_1, \Pi_2, \Pi_3, \ldots \) are dimensionless groups.

Each \( \Pi \) term contains \( (m+1) \) variables out of which \( m \) are of repeating type and one is of non-repeating type.

Each \( \Pi \) term being dimensionless, the dimensional homogeneity can be used to get each \( \Pi \) term.

**Method of Selecting Repeating Variables**

1. Avoid taking the quantity required as the repeating variable.

2. Repeating variables put together should not form dimensionless group.

3. No two repeating variables should have same dimensions.

4. Repeating variables can be selected from each of the following properties
   a. Geometric property - Length, Height, Width, Area
   b. Flow property - Velocity, Acceleration, Discharge
   c. Fluid property – Mass Density, Viscosity, Surface Tension

**Problem 1:** Find an expression for drag force \( R \) on a smooth sphere of diameter \( D \) moving with uniform velocity \( V \) in a fluid of density \( \rho \) and dynamic viscosity \( \mu \).

**Solution:** \( f(R, D, V, \rho, \mu) = 0 \)

Here, \( n = 5, m = 3 \)
Therefore, Number of $\Pi$ terms = $(n - m) = 5 - 3 = 2$

$$f(\Pi_1, \Pi_2, \Pi_3) = 0$$

Let $D, V, \rho$ be the repeating variables.

$$\Pi_1 = D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot R$$

\[
\begin{bmatrix}
L^3 M^0 T^0 \\
L^3 M^0 T^0
\end{bmatrix} =
\begin{bmatrix}
L^1 T^{-1} \\
L^1 T^{-1}
\end{bmatrix}
\begin{bmatrix}
M^{-3} \\
M^{-3}
\end{bmatrix}
\begin{bmatrix}
L^{-1} M T^{-2} \\
L^{-1} M T^{-2}
\end{bmatrix}
\]

$L^0 M^0 T^0 = [L]^{a_1} [T]^{b_1} [M]^{c_1} [T]^{-b_1}$

- $b_1 = 2$
- $c_1 = 0$
- $c_1 = -1$
- $a_1 + b_1 - 3c_1 + 1 = 0$
- $a_1 + 2 + 3 + 1 = 0$

$a_1 = -2$

$$\Pi_1 = D^2 \cdot V^2 \cdot \rho^{-1} \cdot R$$

$$\frac{R}{\Pi_1} = \frac{D^2 V^2 \rho}{F}$$

$$\Pi_2 = D^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot \mu$$

\[
\begin{bmatrix}
L^0 M^0 T^0 \\
L^0 M^0 T^0
\end{bmatrix} =
\begin{bmatrix}
L^2 T^{-1} \\
L^2 T^{-1}
\end{bmatrix}
\begin{bmatrix}
M^{c_2} \\
M^{c_2}
\end{bmatrix}
\begin{bmatrix}
L^{-1} M T^{-1} \\
L^{-1} M T^{-1}
\end{bmatrix}
\]

$L^0 M^0 T^0 = [L]^{a_2 + b_2 - 3c_2 - 1} [M]^{c_2 + 1} [T]^{-b_2 - 1}$

- $b_2 - 1 = 0$
- $b_2 = -1$
- $c_2 = -1$
- $a_2 + b_2 - 3c_2 - 1 = 0$
- $a_2 - 1 + 3 - 1 = 0$
- $a_2 = -1$

$$\Pi_2 = D \cdot V \cdot \rho \cdot \mu$$

$$\Pi_2 = \frac{\mu}{\rho V D}$$

$$f(\Pi_1, \Pi_2) = 0$$
Problem 2: The resisting force of a supersonic plane during flight can be considered as dependent on the length of the aircraft $L$, velocity $V$, viscosity $\mu$, mass density $\rho$, and Bulk modulus $K$. Express the fundamental relationship between resisting force and these variables.

Solution: $f (R, L, K, \mu, \rho, V) = 0$

Therefore, Number of $\Pi$ terms = $6 - 3 = 3$

$f (\Pi_1, \Pi_2, \Pi_3) = 0$

Let, $L$, $V$, $\rho$ be the repeating variables.

$$\Pi_1 = L^{a_1} V^{b_1} \rho^{c_1} K$$

$$L^o M^o T^o = [L]^{a_1} [LT^{-1}]^{b_1} [ML^{-3}]^{c_1} [L^{-1}MT^{-2}]$$

$$L^o M^o T^o = \left[ L \right]^{a_1 + b_1 - 3c_1 - 1} \left[ M \right]^{c_1 + 1} \left[ T \right]^{-b_1 - 2}$$

$$b_1 = -2$$

$$c_1 = -1$$

$$a_1 + b_1 - 3c_1 - 1 = 0$$

$$a_1 - 2 + 3 - 1 = 0$$

$$a_1 = 0$$

$$\Pi_1 = L^o V^{-1} \rho^{-1} K$$

$$\Pi_1 = \frac{V^2}{\rho}$$

$$\Pi_2 = L^{a_2} V^{b_2} \rho^{c_2} R$$

$$L^o M^o T^o = [L]^{a_2} [LT^{-1}]^{b_2} [ML^{-3}]^{c_2} [LMT^{-2}]$$

$$L^o M^o T^o = [L]^{a_2 + b_2 - 3c_2 + 1} [M]^{c_2 + 1} [T]^{-b_2 - 2}$$

$$-b_2 - 2 = 0$$

$$b_2 = -2$$

$$c_2 = -1$$
MODEL STUDIES

Before constructing or manufacturing hydraulics structures or hydraulics machines tests are performed on their models to obtain desired information about their performance. Models are small scale replica of actual structure or machine. The actual structure is called prototype.

Similitude  It is defined as the similarity between the prototype and its model. It is also known as similarity. There three types of similarities and they are as follows.

- Geometric similarity
• Kinematic similarity

• Dynamic similarity

Geometrical Similarity

Geometric similarity is said to exist between the model and prototype if the ratio of corresponding linear dimensions between model and prototype are equal.

\[ \frac{L_p}{L_m} = \frac{h_p}{h_m} = \frac{H_p}{H_m} \ldots \ldots L_r \]

Where, \( L_r \) is known as scale ratio or linear ratio.

\[ \frac{A_p}{A_m} = (L_r)^2, \quad \frac{v_p}{v_m} = (L_r)^3 \]

• Kinematic Similarity

Kinematic similarity exists between prototype and model if quantities such at velocity and acceleration at corresponding points on model and prototype are same.

\[ \frac{(V_1)_p}{(V_1)_m} = \frac{(V_2)_p}{(V_2)_m} = \frac{(V_3)_p}{(V_3)_m} \ldots \ldots V_r \]

Where, \( V_r \) is known as velocity ratio.

• Dynamic Similarity

Dynamic similarity is said to exist between model and prototype if ratio of forces at corresponding points of model and prototype is constant.

\[ \frac{(F_1)_p}{(F_1)_m} = \frac{(F_2)_p}{(F_2)_m} = \frac{(F_3)_p}{(F_3)_m} \ldots \ldots F_r \]

Where, \( F_r \) is known as force ratio.

• Dimensionless Numbers

Following dimensionless numbers are used in fluid mechanics.

1. Reynolds’s number
2. Froude’s number
3. Euler’s number
4. Weber’s number
5. Mach number
1. Reynold’s number

It is defined as the ratio of inertia force of the fluid to viscous force.

\[ N_{Re} = \frac{F_i}{F_v} \]

Expression of Reynolds number \( N_{Re} \)

\( F_i = \text{Mass} \times \text{Acceleration} \)

\( F_i = \rho \times \text{Volume} \times \text{Acceleration} \)

\( F_i = \rho \times Q \times V \)

\( F_i = \rho AV^2 \)

\( F_V \rightarrow \text{Viscous force} \)

\( F_V = \tau \times A \)

\( F_V = \mu \frac{V}{A} \)

\[ N_{Re} = \frac{\rho AV^2}{\mu \frac{V}{A}} \]

In case of pipeline diameter is the linear dimension.

\[ N_{Re} = \frac{\rho VD}{\mu} \]

2. Froude’s Number (\( F_f \))

It is defined as the ratio of square root of inertia force to gravity force.

\[ F_f = \sqrt{\frac{F_i}{F_g}} \]

\( F_i = m \times a \)

\( F_i = \rho \times \text{Volume} \times \text{Acceleration} \)

\( F_i = \rho AV^2 \)

\( F_g = m \times g \)

\( F_g = \rho \times \text{Volume} \times g \)

\( F_g = \rho \times A \times L \times g \)

\[ F_f = \frac{\sqrt{\rho AV^2}}{\sqrt{\rho ALg}} \]
\[ F\gamma = \frac{v}{\sqrt{gL}} \]

**Model Laws (Similarity laws)**

1. **Reynold’s Model Law**

   For the flows where in addition to inertia force, similarity of flow in model and predominant force, similarity of flow in model and prototype can be established if Re is same for both the system.

   This is known as Reynold’s Model Law.

   \[ \text{Re for model} = \text{Re for prototype} \]

   \[ (N_{Re})_m = (N_{Re})_p \]

   \[ \left( \frac{\rho V D}{\mu} \right)_m = \left( \frac{\rho V D}{\mu} \right)_p \]

   \[ \frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p} = 1 \]

   \[ \frac{\rho_r V_r D_r}{\mu_r} = 1 \]

   Applications:
   
   i) In flow of incompressible fluids in closed pipes.
   
   ii) Motion of submarine completely under water.
   
   iii) Motion of air-planes.

2. **Froude’s Model Law**

   When the force of gravity is predominant in addition to inertia force then similarity can be established by Froude’s number. This is known as Froude’s model law.

   \[ (Fr)_m = (Fr)_p \]

   \[ \left( \frac{V}{\sqrt{gL}} \right)_m = \left( \frac{V}{\sqrt{gL}} \right)_p \]

   \[ \left( \frac{V}{\sqrt{gL}} \right)_r = 1 \]

   Applications:
   
   i) Flow over spillways.
ii) Channels, rivers (free surface flows).

iii) Waves on surface.

iv) Flow of different density fluids one above the other.
UNIT 2

UNIFORM FLOW IN OPEN CHANNEL

Introduction

An open channel is a passage in which liquid flows with a free surface, open channel flow has uniform atmospheric pressure exerted on its surface and is produced under the action of fluid weight. It is more difficult to analyze open channel flow due to its free surface. Flow is an open channel is essentially governed by Gravity force apart from inertia and viscous forces.

Comparison between Pipe Flow and Channel Flow

<table>
<thead>
<tr>
<th>Pipe Flow</th>
<th>Channel Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Flow occurs due to difference of pressure.</td>
<td>1. Flow occurs due to the slope of the channel</td>
</tr>
<tr>
<td>2. Free surface is absent in a pipe flow</td>
<td>2. Free surface is present in an open channel flow.</td>
</tr>
<tr>
<td>3. Line joining piezometric surface</td>
<td>3. Liquid surface itself represents the hydraulic grade line (HGL)</td>
</tr>
<tr>
<td>(Z+p/γ) indicates the hydraulic Grade line</td>
<td>4. For uniform flow in an open channel, the drop in the energy gradient line is equal to the drop in the bed.</td>
</tr>
<tr>
<td>4. There is no relation b/w the drop of the energy gradient line and slope of the pipe axis.</td>
<td>5. The total energy line lies at a distance of (V^2/2g) above the H.G.L at every section.</td>
</tr>
<tr>
<td>5. The total energy line lies at a distance of (V^2/2g) above the H.G.L at every section.</td>
<td>6. Pressure difference between two sections cannot be built up.</td>
</tr>
<tr>
<td>6. Pressure difference can be built between two sections.</td>
<td>7. If Reynolds number is less than 500 the flow is Laminar.</td>
</tr>
<tr>
<td>7. If Reynolds number is less than 2000 the flow is Laminar.</td>
<td>If it is between 500-600 the flow is known as transition flow.</td>
</tr>
<tr>
<td>If it is between 2000-4000 the flow is known as transition flow.</td>
<td>If it is less than 2000 the flow is known as turbulent flow.</td>
</tr>
<tr>
<td>If it is less than 4000 the flow is known as turbulent flow.</td>
<td></td>
</tr>
</tbody>
</table>
CLASSIFICATION OF CHANNELS

There are two types of channels
   i) Natural Channel
   ii) Artificial Channel

Natural channels: The irregular sections of varying shapes are the natural channels.
Ex. Rivers, streams and drains etc.

Artificial Channels: Artificial open channels are built for some specific purpose, such as irrigation, water supply, water power development etc. Such channels are regular in shape and alignment. Surface roughness is also uniform.

Depending upon the shape, a channel is either prismatic or non-prismatic.

A channel is said to be prismatic when the cross section is uniform and the bed slope is constant. Ex. Rectangular, trapezoidal, circular, parabolic.

A channel is said to be non-prismatic when its cross section and for slope change.
Ex: River, Streams & Estuary.

Open Channels and Closed Channels: A channel without any cover at the top are known as open channel. A channel having cover at the top is known as closed channel.

Types of flow in open channel
Flow in an open channel can be classified into different types based on different criteria.
   a) Steady Flow and Unsteady Flow: Flow in an open channel is said to be steady when the flow characteristics like depth, discharge, mean velocity at any point do not change with time i.e. \( \frac{\partial v}{\partial t} = 0 \), \( \frac{\partial y}{\partial t} = 0 \) is called steady flow.
      If any of these characteristics change with time then the flow is called as unsteady flow \( \frac{\partial v}{\partial t} \neq 0 \), \( \frac{\partial y}{\partial t} \neq 0 \).
   b) Uniform and Non uniform flow: Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (length of direction of flow). Mathematically \( (\frac{\partial v}{\partial s})_{t=\text{constant}} = 0 \)
      Non uniform flow is that type of flow in which the velocity at any given time changes
with respect to space. Thus mathematically \( \frac{\partial v}{\partial s} \) \(_{t=constant} \neq 0 \)

Non uniform flow in open channel is also called as varied flow, which is classified into following two types.

i) Rapidly varied Flow (R.V.F)

ii) Gradually varied Flow (G.V.F)

Rapidly varied flow: Rapidly varied flow is defined as that type of flow in which depth of flow changes abruptly over a small length of the channel. When there is any obstruction in the path of flow of water, the level of water rises above the obstruction and then falls and again rises over a small length of channel. Thus the depth of flow changes rapidly over a short length of the channel. For this short length of the channel the flow is called rapidly varied flow.

Gradually varied flow: If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow.

c) Laminar and Turbulent flow: Laminar flow is defined as that type of flow in which the fluid particles move along well defined paths or stream line and all the stream lines are straight and type of flow is also called stream line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a zigzag way.

The flow in open channel is said to be laminar if the Reynolds number Re is less than 500 or 600. If the Reynolds number is more than 2000, the flow is said to be turbulent. If Re lies between 500-2000, the flow is said to be in transition state.

d) Sub critical, critical and Super critical flow: The flow in open channel is said to be sub critical if the Froude’s number (Fe) is less than 1.0. Sub critical flow is also called tranquil or streaming flow.

The flow is said to be critical flow if Fe = 1.0 and if the flow is said to be super critical or shooting or rapid or torrential if Fe is greater than 1.0.

**Geometric properties of open channels**

**Depth of flow (y):** It is the vertical distance between the lowest points of the channel sections from the free liquid surface. It is expressed in meters.

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Area of cross section or Wetted area \((A)\) It is the area of the liquid surface when a cross section is taken normal to the direction of flow. It is expressed in \(m^2\).

Wetted perimeter \((P)\): It is the length of the channel boundary in contact with the flowing liquid at any section. It is expressed in meters.

Hydraulics radius or Hydraulic mean depth \((R)\): It is the ratio of area of cross section \((A)\) to the wetted perimeter \((P)\). It is expressed in meters.

\[
R = \frac{A}{P}
\]

Top width \((T)\): It is the width of the channel at the free surface as measured perpendicular to the direction of flow at any given section. It is expressed in meters.

Hydraulic depth \((D)\): It is the ratio of area of cross section \((A)\) to the top width \((T)\).

\[
D = \frac{A}{T}
\]

Section factors \((Z)\): It is the product of the area of cross section \((A)\) to the square root of the hydraulic depth \((D)\).

\[
Z = A\sqrt{D}
\]

Hydraulic Slope \((S)\): Hydraulic slope of the total energy line is defined as the ratio of drop in total energy line \((h_f)\) to the channel length \((L)\).

\[
S = \frac{h_f}{L}
\]

**Geometric properties for different types of prismatic channels**

Rectangular Channel:
Consider a rectangular channel whose width is \(B\) and depth of flow is \(y\) therefore the area of cross section

\[
A = B \times y
\]

Then wetted perimeter

\[
P = B + 2y
\]

Top width \(T=B\)

Hydraulic depth \(D = \frac{A}{T} = \frac{By}{B} = y\)

Trapezoidal channel: Consider a trapezoidal channel, let \(n\) be the side slope, \(B\) be the
bottom width, \( y \) be the depth of flow and \( T \) top width. Therefore the area of the cross section

Area of flow \( A = (\text{Area of rectangular } 2x \text{ Area of the half triangle}) \)

\[
A = (By + (2x (1/2) \times ny \times y))
\]

Wetted perimeter \( P = B + 2 \sqrt{n^2y^2 + y^2} \)

\[
P = B + 2y\sqrt{n^2 + 1}
\]

Top Width \( T = B + 2ny \)

Hydraulic radius, \( R = \frac{A}{P} \)

\[
R = \frac{B + ny^2}{B + 2y\sqrt{1 + n^2}}
\]

Hydraulic Depth, \( D = \frac{A}{T} \)

\[
D = \frac{By + ny^2}{B + 2ny}
\]

Triangular channel: Consider a triangular channel, let the top width be \( T \) and depth of flow be \( y \) and slope \( 1:n \) then

Area of cross section is \( A = ((1/2) \times b \times h) \)

\( A = ny^2 \)

Wetted perimeter \( P = 2\sqrt{n^2y^2 + y^2} \)

\( P = 2y\sqrt{n^2 + 1} \)

Top width \( T = 2ny \)

Hydraulic Radius, \( R = \frac{A}{P} \)

\[
R = \frac{ny^2}{2y\sqrt{1 + n^2}}
\]

\[
R = \frac{ny}{2\sqrt{1 + n^2}}
\]

Hydraulic Depth, \( D = \frac{A}{T} \)

\[
D = \frac{ny^2}{2ny}
\]

\[
D = \frac{y}{2}
\]

Uniform Flow in open channels

Flow in an open channel is said to be uniform when the parameters such as depth area of cross section, velocity discharge etc., remain constant throughout the entire length of the channel.
Features of Uniform flow
a] Depth of flow, area of cross section, velocity and discharge are constant at every section along the channel reach.
b] Total energy line, water surface and channel bottom are parallel to each other, also their slopes are equal or \( S_o = S_w = S_f \)
\[
S_f = \text{energy line slope} \quad S_o = \text{channel bed slope} \quad S_w = \text{water surface slope}
\]

**CHEZY’S FORMULA**

Consider uniform flow between two sections 11 and 2 2, L distant apart as shown

Various forces acting on the control volume are:

i] Hydrostatic forces

ii] Component of weight \( w \sin \theta \) along the flow.

iii] Shear or resistance to flow acting along the wetted perimeter and opposite to the direction of motion

From second law of Newton

Force = Mass x acceleration

As the flow is uniform, acceleration = Zero (O) \( \therefore \sum \text{forces} = 0 \)

\[
forces = + F_1 - F_3 + w \sin \theta - \tau_0 \times \text{contact area} = 0
\]

Again \( F_1 = F_3 \) \( \because \) Flow is uniform

\[
\therefore w \sin \theta - \tau_0 \times \text{contact area} = 0
\]

\[
\therefore w \sin \theta = \tau_0 \times \text{contact area} - (1)
\]

From the definition of specific weight \( \gamma = \frac{weight}{volume} \)

Weight \( w = \gamma \times \text{volume} \)

\[
= \gamma \times A \times L
\]

Contact area = wetted perimeter \( \times \) length = \( P \times L \)

Also, for small values of \( \theta, \sin \theta \approx \tan \theta = 0 \)

Substituting all values in eq 1 and simplifying
Hydraulics and Hydraulic Machines (10CV45)

\[ AE \cdot S_r = \tau_0 \cdot RL \]
\[ \tau_0 = \frac{A}{P} \cdot S_r \]

But, \[ \frac{A}{P} = R (\text{Hydraulic radius}) \]
\[ \therefore \tau_0 = \gamma R S_r \quad (2) \]

From experiment it is established that shear stress \[ \tau_0 = \frac{f}{8} \rho V^2 \]
\[ \therefore \frac{f}{8} \rho V^2 = \gamma R S_r \quad V = \sqrt[4]{\frac{8 \gamma}{\rho_f}} \sqrt{R S_r} \quad \text{or} \]
\[ V = C \sqrt{RS} \]

Where, \[ C = \sqrt[4]{\frac{8 \gamma}{\rho_f}} \]
\[ C = \text{Chezy’s constant} \]

From continuity equation \[ Q = AV \]
\[ \therefore Q = AC \sqrt{R S_r} \quad (3) \]

It should be noted that chezy’s C is not just a non-dimensional number and it has a

dimension of \[ \left[ \frac{1}{L^3 \cdot T^{-1}} \right] \]

Chezy’s equation is used in pipe flow also. The value of Chezy’s C is different for

Different types of channels.

**MANNING’S FORMULA**

Robert Manning in 1889, proposed the formula \[ V = \frac{1}{N} R^{rac{2}{3}} S_r^{rac{1}{2}} \]

The above formula is known as Manning’s formula where \( N \) is Manning’s roughness or rugosity coefficient. Similar to Chezy’s C Table 1 gives the range of value of the

**Manning’s constant N**

<table>
<thead>
<tr>
<th>Sl.no</th>
<th>Surface</th>
<th>Recommended Value of N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glass, Plastic, Brass</td>
<td>0.010</td>
</tr>
<tr>
<td>2</td>
<td>Timber</td>
<td>0.011 – 0.014</td>
</tr>
<tr>
<td>3</td>
<td>Cement plaster</td>
<td>0.011</td>
</tr>
<tr>
<td>4</td>
<td>Cast iron</td>
<td>0.013</td>
</tr>
<tr>
<td>5</td>
<td>Concrete</td>
<td>0.012 – 0.017</td>
</tr>
<tr>
<td>6</td>
<td>Drainage tile</td>
<td>0.013</td>
</tr>
<tr>
<td>7</td>
<td>Brickwork</td>
<td>0.014</td>
</tr>
<tr>
<td>8</td>
<td>Rubble masonry</td>
<td>0.017 – 0.025</td>
</tr>
<tr>
<td>9</td>
<td>Rock cut</td>
<td>0.035 – 0.040</td>
</tr>
</tbody>
</table>

Dept. of Civil Engg., ACE
Most Economical Or Most Efficient Or Best Hydraulic Open Channel

Definition: The most efficient cross section may be defined as that offers least resistance to flow and hence passes maximum discharge for a given slope, area and roughness.

From continuity equation \( Q=AV \), Discharge \( Q \) is maximum when the velocity \( V \) is maximum for a given area of cross section \( A \)

From Chezy’s equation \( V = c\sqrt{RS_0} \) Velocity \( V \) is maximum when the hydraulic radius \( R \) is maximum for given values of Chezy’s \( C \) and bed slope \( S_0 \)

But, by definition Hydraulic radius \( R = \frac{A}{P} \)

Therefore hydraulic radius \( R \) is maximum when the wetted perimeter \( P \) is minimum for a given area of cross section \( A \). Hence an open channel is most economical when the wetted perimeter \( P \) is least or minimum for a given area of cross section \( A \).

MOST ECONOMICAL RECTANGULAR OPEN CHANNEL

Area \( A=By \) .................. (i)

\( B=A/y \) ...................... (ii)

Wetted perimeter \( P=B+2y \) (iii)

Substituting eq (ii) in eq (iii)

\[ P = \frac{A}{y} + 2y \]  (iv)

For the channel to be most economical wetted perimeter should be minimum. But from Eq (iv) we see that \( P \) is a function of the depth of flow \( y \), for a given area of cross section \( A \). Hence for the condition is that \( \frac{dp}{dy} = 0 \)

It may thus be concluded that for a rectangular channel to be most economical or efficient the bed width Should be twice the depth of flow or the hydraulic radius \( R \) should be half the depth of the flow.
MOST ECONOMICAL TRIANGULAR CHANNEL

For a triangular section, if $\theta$ is the angle of inclination angle of each of the sloping sides with the vertical and $y$ is the depth of flow, the following expression for the wetted area $A$ and wetted perimeter $P$ can be written as

$A = ny^2$

$A = \tan \theta y^2$

$y = \frac{A}{\tan \theta}$  \hspace{1cm} (ii)

Substituting the value of $y$ in eqn(i) in(ii) it becomes

$P = \frac{2 \sec \theta \sqrt{A}}{\sqrt{\tan \theta}}$ \hspace{1cm} (iii)

Assuming the area $A$ to be constant eqn (iii) can be differentiated w.r.t $\theta$ and equated to 0 for obtaining the condition for minimum $P$

Thus $\frac{dP}{d\theta} = 2 \sqrt{A} \left( \frac{\sec \theta \tan \theta}{\sec \theta} - \frac{\sec^3 \theta}{2 \tan^2 \theta} \right) = 0$

$\sec \theta (2 \tan^2 \theta - \sec^2 \theta) = 0$

Since $\sec \theta \neq 0$, $(2 \tan^2 \theta - \sec^2 \theta) = 0$

$\sqrt{2} \tan \theta = \sec \theta$

$\theta = 45^0$ or $n = 1$ \hspace{1cm} (iv)

hence a triangular channel section will be most economical or most efficient when each of its sloping sides makes an angle of $45^0$ with the vertical.

The hydraulic radius $R$ of the channel section can be expressed as

$R = \frac{A}{P} = \frac{y^2 \tan \theta}{2y \sec \theta}$, substituting the value of $\theta$

$R = \frac{y}{2\sqrt{2}}$

Thus it can be seen that the most economical triangular channel section will be half square described on a diagonal and having sloping sides.

Most economical Trapezoidal Channel section

The trapezoidal section of a channel will be most economical, when its wetted perimeter is minimum. Consider a trapezoidal section of channel. Let $b$ be the width of the channel bottom, $d$ be the depth of flow and $\theta$ be the.

i) The side slope is given as 1 vertical to $n$ horizontal
Therefore, Area of flow  \( A = (b + (b + 2nd)) x d/2 \)
\[ = (2b + 2nd) x d/2 \]
\[ = (b + 2nd) d \ldots \ldots (i) \]

Therefore, \( (A/d) = b + nd \)
\[ b = (A/d) - nd \ldots \ldots \ldots \ldots (ii) \]

now wetted perimeter, \( P = b + 2\sqrt{n^2d^2 + d^2} \)
\[ = b + 2d\sqrt{n^2 + 1} \ldots \ldots (iia) \]

Substituting the value of \( b \) from equation (ii), we get
\[ P = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \]

For most economical section, \( P \) should be minimum \( \frac{dP}{d(d)} = 0 \)

Differentiating the eqn w.r.t \( d \) and equating it to zero, we get
\[ \frac{d}{d(d)} (\frac{A}{d} - nd + 2d\sqrt{n^2 + 1}) = 0 \]
\[ \frac{A}{d^2} + n = 2\sqrt{n^2 + 1} \]
substituting the value of \( A \) from eqn (i) in the above eqn
\[ \frac{(b + nd)d}{d^2} + n = 2\sqrt{n^2 + 1} \]
\[ b + nd + nd \]
\[ d \]
\[ b + 2nd \]
\[ = d \sqrt{n^2 + 1} \]

But \( \frac{b + 2nd}{2} \) = half the top width and \( d\sqrt{n^2 + 1} \) is one of the sloping sides.

Therefore for a trapezoidal channel to be most economical “half the top width must be equal to one of the sloping sides of the channel”.

(ii) Hydraulic Mean depth, \( R = \frac{A}{P} \)

Substituting the value of \( A \) and \( P \) we get, \( R = \frac{(b + 2nd)d}{2(b + nd)} \)
\[ R = \frac{d}{2} \]
Hence for a trapezoidal channel to be most economical hydraulic mean radius must be equal to half the depth of flow.

(iii) the three sides of thee trapezoidal section of the most economical section are tangential to the semi-circle subscribed on the water linr. This is proved as follows

Let \( \theta \) be the angle made by the sloping side with horizontal and O be the centre of the top width, AD. Draw OF perpendicular to the sloping side AB, triangle OAF is a right angled triangle and angle OAF = \( \theta \)

Therefore \( \sin \theta = \frac{OF}{OA} \), OF = OA \( \sin \theta \), In \( \Delta AEB \), \( \sin \theta = \frac{AE}{AB} \)

\[ \frac{d}{\sqrt{d^2 + n^2}} = \frac{d}{\sqrt{1 + n^2}} = \frac{1}{\sqrt{1 + n^2}} \] \quad \text{(v)}

Substituting \( \sin \theta = \frac{1}{\sqrt{1 + n^2}} \) in eqn (iv) we get,

\[ OF = AO \times \frac{1}{\sqrt{1 + n^2}} \]

But \( AO \) = half of top width = \( \frac{b + 2nd}{2} \)

\[ = d\sqrt{1 + n^2} \]

Substituting in equation (v)

\[ OF = \frac{d\sqrt{1 + n^2}}{\sqrt{1 + n^2}} \]

OF = d, depth of flow thus if a semi circle is drawn with O as centre and radius equal to the depth of flow D the three sides of a most economical trapezoidal section will be tangential to the semi-circle.
UNIT 3
NON UNIFORM FLOW IN OPEN CHANNELS

Introduction

Non uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus mathematically \( \frac{\partial v}{\partial s} |_{t=constant} \neq 0 \)

Non uniform flow in open channel is also called as varied flow, which is classified into following two types.

i) Rapidly varied Flow (R.V.F)

ii) Gradually varied Flow (G.V.F)

SPECIFIC ENERGY (E)

The concept of specific energy was introduced by BORIS A BACK METEFF (1912). It is a very useful concept in the study of open channel flow problems.

Specific energy \( E \) is defined as the energy per unit weight of the liquid at a cross section measured above the bed level at that point.

The total energy of the following fluid per unit weight is given by

\[
\text{Total Energy} = Z + h + \frac{v^2}{2g}
\]

Where \( Z = \) Height of the bottom of channel above datum,

\( h = \) Depth of Liquid

\( V = \) Mean Velocity of Flow

If the channel bottom is taken as the datum, then the total energy per unit weight of liquid will be

\[ E = h + \frac{v^2}{2g} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (i) \]
Specific Energy Curve: It is defined as the curve which shows the value of specific energy with the depth of flow. It is obtained as from eqn (i), the specific energy of a flowing liquid

\[ E = h + \frac{v^2}{2g} = E_p + E_k \]

Where, \( E_p = \) potential Energy of flow = \( h \)

\[ E_k = \text{Kinetic Energy of flow} = \frac{v^2}{2g} \]

Consider a rectangular channel in which a steady but non uniform flow takes place.

Let \( Q \) be the discharge through the channel, \( b \) be the width of the channel, \( h \) be the depth of flow, \( q \) be the discharge per unit width

Then \( q = \frac{Q}{\text{Width}} = \frac{Q}{b} = \text{constant} \)

Velocity of flow, \( V = \text{Discharge} / \text{Area} \)

\[ = \frac{Q}{bxh} \]

\[ = \frac{q}{h} \]

Substituting the values of \( V \) in eqn 1 we get

\[ E = h + \frac{q^2}{2gh^2} = E_p + E_k \ldots \ldots \text{(ii)} \]

Equation (ii) gives the variation of specific energy \( E \) with the depth of flow \( h \). Hence for a given discharge \( Q \), for different values of depth of flow, the corresponding values of \( e \) maybe obtained. Then the graph between specific energy (x axis) and depth of flow \( h \) (y axis) may be plotted.

The specific energy curve may also be obtained by first drawing a curve for the potential energy which will be a straight line passing through the origin making an angle of 45° with the x axis. then drawing another curve for K.E which will be a parabola. By combining the two, we obtain the specific energy curve.
Critical Depth (hc): Critical depth is defined as that depth of flow of water at which the specific energy is minimum. Thus is denoted by hc. The depth of flow of water at C is known as critical depth in specific energy curve. The mathematical expression for critical depth is obtained by differentiating the specific energy eqn (ii) w.r.t depth of flow.

\[ \frac{dE}{dh} = 0 \text{ where } E = h + \frac{q^2}{2gh^2} \]

\[ \frac{d}{dh} \left( h + \frac{q^2}{2gh^2} \right) = 0 \]

\[ 1 + \frac{q^2}{2g} \left( -\frac{2}{h^3} \right) = 0 \]

\[ 1 - \frac{q^2}{gh^3} = 0 \]

\[ 1 = \frac{q^2}{gh^3} \]

\[ h^3 = \frac{q^2}{g} \]

\[ h^3 = \left( \frac{q^2}{g} \right)^{\frac{1}{3}} \]

But when specific energy is minimum, depth is critical and it is denoted by hc. Hence critical depth is

\[ h_c = \left( \frac{q^2}{g} \right)^{\frac{1}{3}} \]

CRITICAL FLOW:

It is defined as the flow at which the specific energy is minimum or the flow corresponding to critical depth is defined as critical flow from the equation

\[ V_c = \sqrt{gh_c} \]
\[ \frac{V_c}{\sqrt{gh_c}} = 1 \]

Therefore Froude number \( F_e = 1.0 \) for critical flow.

STREAMING or SUB CRITICAL or TRANQUIL FLOW: when the depth of flow in a channel is greater than the critical depth \( (h_c) \), the flow is said to be sub critical flow. Froude’s number is less than 1.0 for this type of flow.

SUPER CRITICAL FLOW: when the depth of the flow is less than the critical depth \( h_c \), the flow is said to be super critical flow or torrential or shooting flow. Froude’s number is greater than 1.0 for this type of flow.

CRITERION OF CRITICAL DEPTH \( (h_c) \)

From the foregoing discussion it is evident that the critical depth can be used as a parameter for identifying the flow is sub critical, critical or supercritical. This condition can be obtained by differentiating eq(iii) under the following two headings.

(i) Condition for minimum specific energy \( (E_{min}) \) for a given discharge \( Q \)

from eq (iii) For a given discharge \( Q \) specific energy \( E \) is minimum when

\[ \therefore A \text{ is also a function of } y \]

\[ \therefore \frac{\partial E}{\partial y} = 0 \text{ given } \frac{\partial}{\partial y} \left( y + \frac{Q^2}{2gA^2} \right) = 0 \]

Differentiating and simplifying

\[ \left\{ 1 + \frac{Q^2}{2gA^2} \frac{\partial (A^{-1})}{\partial y} \right\} = 0 \]

\[ 1 + \frac{Q^2}{2gA^2} \left( -2A^{-2} \right) \frac{\partial A}{\partial y} = 0 \]

\[ 1 - \frac{Q^2}{gA^2} \frac{\partial A}{\partial y} = 0 \]

The term \( \frac{\partial A}{\partial y} \) represents the rate of increase of area with respect to the depth \( y \)

\[ \frac{\partial A}{\partial y} = T (\text{Top Width}) \quad \text{Hence} \quad 1 - \frac{Q^2}{gA^2} T = 0 \quad \Rightarrow \quad \frac{Q^2}{gA^2} = 1, \quad \frac{Q^2}{g} = \frac{A^3}{T} \quad \ldots (iv) \]

From continuity equation \( Q = AV \) substituting the value of \( Q \) in Eq(iv)

\[ \frac{A^2V^2}{g} = \frac{A^3}{T} \]

\[ \frac{V^2}{g} = \frac{A}{T} \quad \text{But, } \frac{A}{T} = D \quad (\text{Hydraulic depth}) \]

\[ \frac{V^2}{g} = D \quad \left( \frac{V}{\sqrt{gd}} \right) = 1 \quad \text{or} \quad \frac{V}{\sqrt{gd}} = 1 \]

But, by definition we know that \( \frac{V}{\sqrt{gd}} = F \)
Hence for critical flow Froude number should be unity.

(ii) Condition for max discharge (Qmax) for a given specific energy

From Eq (iii) \( E = y + \frac{Q^2}{2gA^2} \)  

\( Q = \left[ (E - y - \frac{1}{2}gA^2) \right]^{\frac{1}{2}} \)

or, \( Q = A\sqrt{\frac{2g}{E - y}} \)  

For a given specific energy \( E \), discharge \( Q \) is maximum when \( \frac{\partial Q}{\partial y} = 0 \)

A is also a function of \( y \)

Differentiating Eq(vi) w.r.t. \( y \) and equating to zero \( \frac{\partial Q}{\partial y} = 0 \)

\[ i.e., \frac{\partial}{\partial y} \left[ A\left( E - y \right)^{\frac{1}{2}} \right] = 0 \]

Differentiating by parts,

\[ \left\{ -\frac{A}{2} \left( E - y \right)^{\frac{1}{2}} \right\} + \left( E - y \right)^{\frac{1}{2}} \frac{\partial A}{\partial y} = 0 \]

\[ \frac{1}{2} \frac{A}{2(E - y)^{\frac{1}{2}}} \]

As mentioned earlier

\[ hence, \left( \frac{A}{2} \right)^{\frac{1}{2}} = T(E - y)^{\frac{1}{2}} \quad or, \quad E = y + \frac{A}{2T} \]  

Equating Eq(iii) and (vii)

\[ y + \frac{Q^2}{2gA^2} = y + \frac{A}{2T} \quad or, \quad \frac{Q^2}{gA^2} = \frac{A}{T} \quad \therefore \quad \frac{Q^2}{g} = \frac{A^3}{T} \]

This condition is same as Eq(iv) in the previous case i.e., for the condition of minimum specific energy.

Also, the above condition leads to Froude number \( F=1 \)

It may thus be concluded that the conditions for minimum specific energy or maximum discharge, result in the same answer.

Answer, that the Froude number \( F=1 \)

In other words, for critical flow to occur

a) Specific energy \( E \) is minimum for a given discharge \( Q \)
b) Discharge $Q$ is maximum for a given specific energy $E$

c) Froude number $F = 1$ (unity)

But, by definition we know that

$$\frac{v}{\sqrt{gD}} = F$$

Hence for critical flow Froude number should be unity.

From Eq (iii) \( E = y + \frac{Q^3}{2gA^2} \) \((iii)\)

\[ Q = \left( (E - y) \frac{1}{2gA^2} \right)^{1/2} \quad \text{or} \quad Q = A \sqrt{2g} \times \sqrt{E - y} \quad \text{(iv)} \]

For a given specific energy $E$, discharge $Q$ is maximum when $\frac{\partial Q}{\partial y} = 0$

A is also a function of $y$

Differentiating Eq(vi) w.r.t. $y$ and equating to zero. $\frac{\partial Q}{\partial y} = 0$

\[ i.e, \frac{\partial}{\partial y} \left[ A \sqrt{E - y} \right] = 0 \]

Differentiating by parts.

\[ \left\{ \frac{1}{2} (E - y)^{1/2} - (-1) + \frac{1}{2} (E - y)^{1/2} \frac{\partial A}{\partial y} \right\} = 0 \]

\[ \frac{-A}{2(E - y)^{1/2}} + (E - y)^{1/2} T = 0 \]

\[ \therefore \frac{\partial A}{\partial y} = T. \text{ As mentioned earlier} \]

\[ \text{hence,} \quad \frac{A}{2(E - y)^{1/2}} = T(E - y)^{1/2}, \quad \frac{A}{2T} = (E - y) \quad \text{or} \quad E = y + \frac{A}{2T} \quad \text{(vii)} \]

Equating Eq(iii) and (vii)

\[ y + \frac{Q^2}{2gA^2} = y + \frac{A}{2T} \quad \text{or} \quad \frac{Q^2}{gA^2} = \frac{A}{T} \quad \therefore \frac{Q^2}{g} = \frac{A^3}{T} \]
\[ \frac{Q}{B} = q \] as the discharge per unit width of the channel.

\[ T = B, A = By_c, E = E \text{ min}, y = y_c \text{ and } \frac{Q}{B} = q \]

In eq(iv) the condition for critical flow i.e
\[ \frac{Q^2}{g} = \frac{A^3}{T}, \quad \frac{Q^2}{g} = \frac{B^3 y_c^3}{B}, \quad \frac{Q^2}{g} = \frac{B^2 g}{B} \]

\[ \frac{q^2}{g} = y_c^3 \quad \text{Or the critical depth} \quad y_c = \left( \frac{q^2}{g} \right)^{\frac{1}{3}} \quad (ix) \]

Now from the equation for specific energy \[ E = y + \frac{Q^2}{2gA^2} \quad (iii) \]

Substituting the values corresponding to the critical flow.
\[ E \text{ min} = y_c + \frac{Q^2}{2gB^3 y_c^2} = y_c + \frac{q^2}{2gy_c^2} \quad (\because \frac{Q}{B} = q) = y_c + \frac{y_c^3}{2y_c^2} \quad (\because \frac{q^2}{g} = y_c^3) \]

\[ = y_c + \frac{y_c}{2} \quad : E \text{ min} = \frac{3}{2} (y_c) \quad (x) \]

by \[ \frac{\partial q}{\partial y} = 0 \quad i.e, q^2 = 2g(E - y)y^2 \quad \text{or}, \quad q = \{2g(E - y)y^2\} \quad (xi) \]

Differentiating Eq (xi) with respect to y and equating to zero.
\[ \frac{\partial}{\partial y}\left[ y(E - y)^{\frac{1}{2}} \right] = 0 \quad \therefore \sqrt{2g} \neq 0 \]

Differentiating by parts
\[ \{ y \left(\frac{1}{2}(E - y)^{-\frac{1}{2}} \right) (-1) + (E - y)^{\frac{1}{2}} \} = 0 \text{ or } \left\{ \frac{-y}{2(E - y)^{\frac{1}{2}}} + (E - y)^{\frac{1}{2}} \right\} = 0 \]

\[ \text{or, } y = 2(E - y) \]

\[ \text{or, } E = \frac{3}{2} y \]

\[ \text{However, at critical conditions } E = \text{Emin and } y = y_c \]

\[ \text{Emin} = \frac{3}{2} y_c \]

Hence we see that for the critical flow to occur both the conditions converge to the same answer \[ \text{Emin} = \frac{3}{2} y_c \]
HYDRAULIC JUMP

Hydraulic jump is the most commonly encountered varied flow phenomenon on an open channel in which a rapid change occurs from a high velocity low depth super critical state of flow to a low velocity large depth subcritical state.

PLACES OF OCCURRENCE:

a) At the foot of an overflow spitway dam
b) Behind a dam on a steep slope
c) Below a regulating sluice
d) When a steep slope channel suddenly turns flat.

Whenever an hydraulic jump occurs There will be heavy amount of turbulence and considerable energy loss. Hence energy principle or Bernoulli’s energy equation cannot be used for its analysis. Therefore the momentum equation derived from the second law of Newton is used.

USES OF HYDRAULIC JUMP
1. To dissipate excessive energy.
2. To increase the water level on the downstream side.
3. To reduce the net uplift force by increasing the weight, i.e. due to increased depth.
4. To increase the discharge from a sluice gate by increasing the effective head causing flow.
5. To Provide a control section.
6. For thorough mixing of chemicals in water.
7. For aeration of drinking water.
8. For removing air pockets in a pipe line.

Types of Hydraulic Jump (USBR classification)

Based on the initial Froude number F1, Hydraulic Jumps can be classified as follows.
a. Undular Jump Such a jump occurs when the initial Froude number F1 is between 1 and 1.7. In such a jump there will be surface undulations due to low level turbulence it would result in insignificant energy losses.
b. **Weak Jump**: Such a jump occurs when F1 is between 1.7 and 2.5. Head loss is low. In this type of jump, series of small rollers form on the surfaces and the loss of energy due to this type is small.

c. **Oscillating Jump**: It occurs when F1 is between 2.5 and 4.5. In this type the surface will be wavy, jets if water shoot from the floor to surface. Jump moves back and forth causing some damage. Such a jump should be avoided if possible.

d. **Steady Jump**: It occurs when F1, is between 4.5 and 9. Such a jump is stable, balanced in performance, requires a stilling basin to confine the jump. Energy dissipation will be high of the order of 45 to 70%.

e. **Strong Jump**: It occurs when F1 is more than 9. It will be rough and violent, huge rollers are formed in the flow. Energy dissipation will be very high & is upto 85%.

**Analysis of Hydraulic Jump**

The equation of Hydraulic jump can derived making the following assumptions.

1. The channel bed is horizontal so that the component of the body of water weight in the direction of flow can be neglected.
2. The frictional resistance of the channel in the small length over which the jump occurs is neglected, so that the initial and final specific forces can be equated.
3. The channel is rectangular in section.
4. The portion of channel in which the hydraulic jump occurs is taken as a control volume. It is assumed that just before and just after the control volume, the flow is uniform and pressure distribution is hydrostatic.
5. The momentum correction factor (B) is unity.
Consider a hydraulic jump occurring between two sections (1) and (2) as shown.

The various forces acting on the control volume are:

a) Hydraulic pressures forces $F_1$ & $F_2$.

b) Component of weight $W \sin \theta$ in the direction of flow.

c) Shear stress or Frictional resistance acting on the contact area.

From the Impulse momentum equation, the algebraic sum of the forces acting on the control volume is equal to the change in momentum on the control volume.

Consider LHS of equation (1)

forces = $+ F_1 - F_2 + W \sin \theta - \tau_0 \times contact\ area$

As per the assumptions made above, the slope of the channel $\theta$ is very small, i.e., $\sin \theta \approx 0 \therefore W \sin \theta \approx 0$

Hence it can be neglected.

The channel is smooth so that $\theta - \tau_0 \times contact\ area$ can be neglected.

$\therefore \sum forces = F_1 - F_2$

But $F_1$ and $F_2$ being Hydrostatic forces we have $F_1 = \gamma A_1 y_1$, $F_2 = \gamma A_2 y_2$

or, $\sum forces = \gamma A_1 y_1 - \gamma A_2 y_2$

Where $A_1$ and $A_2$ are area of cross section before and after the jump.

$y_1$ and $y_2$ are the centroidal depths $F_1$ and $F_2$, measures from the respective liquid surfaces.

Consider RHS of Eq(1)

Change in momentum = (Momentum before the jump per unit time or Momentum after the jump)

$= \frac{mass}{time} x(velocity \ before \ \approx \ velocity \ after) = \frac{mass}{time} x(velocity \ before \ \approx \ velocity \ after)$
\[ \Delta \text{volume} \left( V_1 \approx V_2 \right) = \ell \times \text{discharge} \left( V_1 \approx V_2 \right) = \ell \cdot Q \left( \frac{Q}{A_1} \approx \frac{Q}{A_2} \right) = \ell \cdot Q \left( \frac{1}{A_1} \approx \frac{1}{A_2} \right) \]

Change in momentum per unit time \[ = \frac{\gamma}{g} \cdot Q^2 \left( \frac{1}{A_1} \approx \frac{1}{A_2} \right) \] (b)

Substituting eq (a) and (b) in eq (1) \[ \gamma \Delta y \approx \gamma \Delta y = \frac{\gamma}{g} \cdot Q^2 \left( \frac{1}{A_1} \approx \frac{1}{A_2} \right) \]

Rewriting \[ \frac{\Delta y_1}{A_1} + \frac{Q^2}{gA_1} = \frac{\Delta y_2}{A_2} + \frac{Q^2}{gA_2} \] (2)

Eq (2) is the general equation of Hydraulic jump in any type of channel.
For a rectangular channel \( A_1 = B y_1, A_2 = B y_2 \)
\[
\tilde{y}_1 = \frac{y_1}{2}, \tilde{y}_2 = \frac{y_2}{2} \text{ and } \frac{Q}{B} = q
\]

Substituting all these values in Eq(2)
\[
B \tilde{y}_1 y_1^2 - 2 \frac{Q^2}{g B^2 y_1} + \frac{Q^2}{g B^2 y_2} = \frac{y_2}{2} + \frac{Q^2}{g B^2 y_2}
\]
\[
\frac{y_1^2}{2} - \frac{Q^2}{g B^2 y_1} = \frac{y_2^2}{2} + \frac{Q^2}{g B^2 y_2} \quad \text{or} \quad \frac{y_1^2}{2} - \frac{Q^2}{g B^2 y_1} = \frac{y_2^2}{2} + \frac{Q^2}{g B^2 y_2}
\]
\[
\frac{2q^2}{g} = \frac{y_1 y_2}{y_1 + y_2} \quad -----(3)
\]

Eq(3) is the general equation of hydraulic jump in a rectangular channel. It can be written as
\[
y_1^2 y_2 + y_1 y_2^2 - \frac{2q^2}{g} = 0 \quad -----(4)
\]

Eq(4) can be quadratic in \( y_1 \) or \( y_2 \). Consider Eq(4) to be quadratic in \( y_1 \)
\[
y_2 y_1^2 + y_1 y_2^2 - \frac{2q^2}{g} = 0
\]
\[
\therefore \ y_1 = \frac{-y_2 \pm \sqrt{y_2^2 - 4y_2 \left( \frac{2q^2}{g} \right)}}{2y_2} = \left\{ \begin{array}{l}
- \frac{y_2}{2} + \frac{\sqrt{y_2^2 + 4q^2 y_2}}{2y_2} \\
- \frac{y_2}{2} - \frac{\sqrt{y_2^2 + 4q^2 y_2}}{2y_2}
\end{array} \right\}
\]
\[
= \left\{ \begin{array}{l}
\frac{-y_2}{2} + \frac{\sqrt{y_2^2 + 4q^2 y_2}}{2y_2} \\
\frac{-y_2}{2} - \frac{\sqrt{y_2^2 + 4q^2 y_2}}{2y_2}
\end{array} \right\} \quad -----(5)
\]

Similarly considering Eq(4) to be quadratic in \( y_2 \), we have
\[
y_2 = \frac{y_1}{2} \left\{ \frac{-1 + \sqrt{1 + \frac{8q^2 y_1}{y_2^3}}}{1 + \frac{8q^2 y_1}{y_2^3}} \right\} \quad -----(6)
\]

Consider the term \( \frac{q^2}{g y_1^2} \) in Eq(5)

For a rectangular channel \( \frac{q^2}{g} = y_1 \)

Eq(5) can be written as
\[
y_1 = \frac{y_1}{2} \left\{ \frac{-1 + \sqrt{1 + \frac{8y_1^3}{y_1^3}}}{1 + \frac{8y_1^3}{y_1^3}} \right\} \quad -----(7)
\]

Similarly from Eq(6) \( y_2 = \frac{y_1}{2} \left\{ \frac{-1 + \sqrt{1 + \frac{8y_1^3}{y_1^3}}}{1 + \frac{8y_1^3}{y_1^3}} \right\} \quad -----(8)
\]

Consider the term \( \frac{q^2}{g y_1^2} \) in Eq(4)
\[
\frac{q^2}{g y_1^2} = \frac{Q^2}{B^2 g y_1^2} = \frac{A_1 y_1^2}{B^2 g y_1^2} = \frac{B^2 y_1^2 y_2^2}{B^2 g y_2^2} = \frac{V_1^2}{g y_2} = \left[ \frac{V_2}{\sqrt{g y_2}} \right]^2
\]
But, \[ \frac{V_2}{\sqrt{g y_2}} = F_2 \text{(Froude number after the jump)} \]

\[ \therefore \frac{q^2}{g y_2^3} = F_2^2 \]

Hence, from Eq(4) \[ y_1 = \frac{y_2}{2} \left[ -1 + \sqrt{1 + 8 F_2^2} \right] \quad (9) \]

Again from Eq(5) \[ y_2 = \frac{y_1}{2} \left[ -1 + \sqrt{1 + 8 F_1^2} \right] \quad (10) \]

In Eq(10) F1 = Froude number before the jump. Eq(9) can be written as

\[ \frac{y_1}{y_2} = \frac{1}{2} \left[ -1 + \sqrt{1 + 8 F_2^2} \right] \quad (11) \]

Similarly Eq(10) as 

\[ \frac{y_2}{y_1} = \frac{1}{2} \left[ -1 + \sqrt{1 + 8 F_1^2} \right] \quad (12) \]

In Eq(11) and (12) \( \frac{y_1}{y_2} \) or \( \frac{y_2}{y_1} \) is known as the ratio of conjugate depths.
UNIT 4
IMPACT OF JETS ON FLAT VANES

Newton’s II Law of Motion:
“The rate of change of momentum of a moving body is directly proportional to
the magnitude of the applied force and takes place in the direction of the applied
force”.

\[ F = \frac{\alpha (mV - mU)}{t} \]

\( \alpha \) ma

a – acceleration

\( F = k \) ma

If \( m = 1 \) and \( a = 1 \) then \( F = 1 \)

\( k = 1 \)

\( F = ma \)

SI unit of force: newton (N)

- Momentum:

  The capacity of a moving body to impart motion to other bodies is called
  momentum.

  The momentum of a moving body is given by the product of mass and
  velocity of the moving body.

  \[ \text{Momentum} = \text{Mass} \times \text{Velocity} \]
  \[ = m \times V \]

  Unit: kgm/s

- Impulsive Force and Impulse of Force:

  A force acting over a short interval of time on a body is called impulsive
  force. Eg: Kick given to a foot ball.

  Impulse of a force is given by the product of magnitude of force and its time
  of action.

  Impulse of a force = Force \times \text{Time interval}
SI unit: Ns

- **Impulse – Momentum Principle:**
  From Newton’s II Law
  
  \[ F = ma \]
  \[ F = m \frac{(V-U)}{t} \]
  \[ F_t = m(V_m - U_m) \]
  
  Impulse = Final momentum – Initial momentum
  
  \[ \therefore \text{Impulse of a force is given by the change in momentum caused by the force on} \]
  \[ \text{the body.} \]
  \[ F_t = m \times \text{Final velocity} - m \times \text{Initial velocity} \]
  \[ F_x t = m (V_x - U_x) \]
  \[ F_y t = m (V_y - U_y) \]

- **Rate of Flow or Discharge:**
  The quantity of fluid flowing across a section in unit time is called rate of flow.
  
  (i) **Volume Flow Rate** (Q):
  It is the volume of fluid flowing across the section in unit time.
  
  Unit: m³/s or cumecs
  
  1000 litre/sec = 1 m³/s
  
  \[ Q = \text{Area of section} \times \text{velocity of flow} \]
  \[ Q = AV \]

  (ii) **Mass Flow Rate** m:
  It is the mass of fluid flowing across the section in unit time.
  Unit: kg/s

  (iii) **Weight Flow Rate** w:
  It is the weight of fluid flowing across a section in unit time.
  Unit: N/s

- **Jet of Liquid**
A nozzle is a tube of reducing cross section. As the water under pressure in the pipe passes through the nozzle, the area of cross section of flow decreases leading to increase in velocity and decreases in pressure. The jet of liquid comes out to the atmosphere.

**Vanes**: Vanes or blades are plates of definite geometrical shape mounted on the periphery of rotor of a turbo machine (Pump / Turbine) in order to transfer energy from rotor to fluid or fluid to rotor.

\[
\begin{align*}
\sum F_x &= \frac{m}{t} (V_x - U_x) \\
\sum F_z &= m(V_z - U_z) \\
\sum F_y &= m(V_y - U_y)
\end{align*}
\]

Where \( m \) is the mass flow rate
\[ m = \rho Q \]
\( V \) = Final velocity of fluid along the direction.
\( U \) = Initial velocity of fluid along the direction.

- **Force Exerted by Jet on Plates**

**Case-I**
To compute the impact of field jet on stationary flat plate held normal to the jet.
V – Velocity of jet striking the plate
a – Area of cross section of jet.

\[ m = \rho a V \]

Force exerted by plate on fluid jet along x – direction

\[ F_x = m (V_x - U_x) \]

Force exerted by the jet on the plate along x – direction will be equal and opposite to that of force exerted by plate on the jet.

\[ \therefore \text{Force exerted by jet on plate along } x \text{ – direction} = F_x = m (V_x - U_x) \]

\[ F_x = \rho a V [V - 0] \]
\[ F_x = \rho a V^2 \]

Work done by the jet = Force x Velocity of plate

\[ = \text{Force} \times 0 \]
\[ = 0 \]

**Case-II**
To compute the impact of jet on a stationary flat plate held inclined to the direction of jet.
Force exerted by jet on vane along normal direction.

\[ F_n = m[U_n - V_n] \]

\[ F_n = \rho aV [(V \sin \theta) - 0] \]

\[ F_n = \rho aV^2 \sin \theta \]

\[ F_x = F_n \cos (90 - \theta) \]

\[ F_x = [\rho aV^2 \sin \theta] \sin \theta \]

\[ F_x = \rho aV^2 \sin^2 \theta \]

\[ F_y = F_n \sin (90 - \theta) \]

\[ F_y = [\rho aV^2 \sin \theta] \cos \theta \]

\[ F_y = \rho aV^2 \sin \theta \cos \theta \]

Work done by the jet on vane

= Force x Velocity of vane

= Force x 0

= 0
Case-III
To compute the impact of jet on a moving flat plate held normal to the jet.

\[ V = \text{Velocity of jet striking the plate} \]
\[ U = \text{Velocity of vane along the direction of vane.} \]

Adopting the concept of relative velocity, the system can be considered to be a stationary plate, the jet striking the vane with a relative velocity \((V - U)\).

\[ \therefore m = \rho Q \]
\[ = \rho a(V - U) \]
\[ F_x = \rho a(V - U)^2 \]

Work done by the jet on plate = Force x Velocity of plate
\[ = \rho a (V - U)^2 x U \]

Case-IV
To compute the impact of jet on a moving flat plate held inclined to the direction of jet.

\[ V = \text{Velocity of jet} \]
\[ U = \text{Velocity of plate along the direction of jet.} \]
Adopting the concept of relative velocity, the above case can be considered to be fixed vane with a jet velocity of \((V - U)\).

\[
\therefore F_n = \rho a (V - U)^2
\]

\[
F_x = \rho a (V - U)^2 \sin \theta
\]

\[
F_y = \rho a (V - U)^2 \sin \theta \cos \theta
\]

Work done by the jet on vane plate along \(x\) – direction

\[= F_x \times \text{Velocity of plate along } x \text{ – direction}\]
\[= \rho a (V - U)^2 \sin^2 \theta U\]

**Expressions for the force exerted, work done and efficiency of impact of jet on a series of flat vanes mounted radially on the periphery of a circular wheel.**

Let us consider flat vanes mounted radially on the periphery of a circular wheel. 
\(V\) is the velocity of jet and ‘\(U\)’ is the velocity of vane. The impact of jet on vanes will be continuous since vanes occupy one after another continuously.

\[
F_x = m (U_x - V_x)
\]
\[ F_x = \rho a V [(V - U) - 0] \]

\[ F_x = \rho a V (V - U) \]

Work done/s or Power = \(F_x \cdot U\)

**Power = \(\rho a V (V - U) U\)**

Efficiency = \(\eta = \frac{\text{Input}}{\text{Output}}\)

\[ \eta = \frac{\rho a V (V - U) U}{\frac{1}{2} (\rho a V) V^2} \]

\[ \eta = \frac{2(V - U) U}{V^2} \]

**Condition for maximum efficiency:**

\[ \eta = \frac{2}{V^2} (V U - U^2) \]

For maximum efficiency

\[ \frac{d\eta}{dU} = 0 \]

\[ \frac{d\eta}{dU} = 0 = \frac{2(V - U) U}{V^2} \]

\[ \therefore U = \frac{V}{2} \]

\[ \therefore \text{Efficiency is maximum when the vane velocity is 50\% of velocity of jet.} \]

\[ \eta_{\text{max}} = \frac{2(V - \frac{V}{2}) \frac{V}{2}}{V^2} \]

\[ = \frac{1}{2} \]

\[ = 50\% \]
UNIT 5

IMPACT OF JETS ON CURVED VANES

a) Jet strikes the curved vane or plate at the center.
Let a jet of water strike affixed plate at the center as shown in fig. the jet after striking the plate comes out with the same velocity if the plate is smooth and there is no loss of energy due to the impact of jet, in the tangential direction of the curved vane. The velocity at the outlet of the plate can be resolved into two components, one in the direction of the jet and the other perpendicular to the direction of the jet.

-ve sign is taken as the velocity at the outlet is in opposite direction of the jet of water coming out from nozzle.

Component of velocity perpendicular to the jet = $V \sin \theta$

Force exerted by the jet in the direction of jet $F_x = \text{mass per sec } [V_{1x} - V_{2x}]$
Where $V_{1x} = \text{initial velocity in the direction of the jet} = V$

$V_{2x} = \text{final velocity in the direction of jet} = -V \cos \theta$

$F_x = \rho a V [V - (-V \cos \theta)] = \rho a V [V + V \cos \theta]$

$F_x = \rho a V^2 [1 + \cos \theta]$

Similarly $F_y = \text{mass per second } [V_{1y} - V_{2y}]$

Where $V_{1y} = \text{initial velocity in the direction of } y = 0$

$V_{2y} = \text{final velocity in the direction of } y = V \sin \theta$
\[ F_y = \rho a V [0 - V \sin \theta ] \]
\[ F_y = - \rho a V^2 \sin \theta \]

-ve sign means that force is acting in the downward direction. In this case the angle of deflection of the jet = \((180^\circ - \theta)\)

**Impact of jet on a moving symmetrical curved plate, the jet striking the plate at its centre.**

![Diagram of jet impact on a curved plate](image)

Adopting relative velocity concept, the system can be considered to be a jet of relative velocity \((V - U)\) striking a fixed plate.

\[ m = \rho a (V - U) \]

\[ F_x = m [Ux - Vx] \]
\[ F_x = \rho a (V - U) [(V - U) - (-V - U) \cos \theta] \]
\[ F_x = \rho a (V - U)^2 (1 + \cos \theta) \]

Work done by the jet on plate = Force x Velocity of date
\[ = F_x U \]
\[ = \rho a(V - U)^2 (1 + \cos \theta) U \]
The impact of jet on a stationery symmetrical curved plate, the jet striking the plate at one of the tips tangentially.

\[ m = \rho a V \]

\[ F_x = m \left[ U_x - V_x \right] \]
\[ F_x = \rho a V \left[ V \cos \theta - (-V \cos \theta) \right] \]
\[ F_x = \rho a V^2 \left( 1 + \cos \theta \right) \]
\[ F_y = m \left[ U_y - V_y \right] \]
\[ F_x = \rho a V \left[ V \sin \theta - V \sin \theta \right] \]
\[ F_x = 0 \]

Work done by the jet on plate is zero since the plate is stationery.

**Expression for the force exerted, power and efficiency of impact of jet on a series of symmetrical curved vanes mounted on the periphery of a wheel.**

\[ F_x = (\rho a V) \left( U_x - V_x \right) \]
Fx = \( \rho a V \left[ (V - U) + (V - U) \cos \theta \right] \)

\[ F_x = \rho a V \left( V - U \right) \left( 1 + \cos \theta \right) \]

Power = work done/s

Power = \( F_x \times U \)

\[ \text{Power} = \rho a V \left( V - U \right) \left( 1 + \cos \theta \right) U \]

Input = \[ \frac{1}{2} \text{m} V^2 \]

Input = \[ \frac{1}{2} \rho a V^3 \]

\[ \eta = \frac{\text{input}}{\text{output}} \]

\[ \eta = \frac{\rho a V \left( V - U \right) \left( 1 + \cos \theta \right) U}{\frac{1}{2} \rho a V^3} \]

\[ \eta = \frac{2(V - U)(1 + \cos \theta)U}{V^2} \]
Condition for maximum efficiency

\[ \eta = \frac{2(VU - U^2)(1 + \cos \theta)}{V^2} \]

For maximum efficiency

\[ \frac{d\eta}{dU} = 0 = \frac{2(V - 2U)(1 + \cos \theta)}{V^2} \]

\[ V - 2U = 0 \]

\[ V = 2U \]

\[ U = \frac{V}{2} \]

Vane velocity = \( \frac{1}{2} \) x velocity of jet

\[ \eta_{max} = \frac{2 \left( V - \frac{V}{2} \right) (1 + \cos \theta) \frac{V}{2}}{V^2} \]

\[ \eta_{max} = \frac{(1 + \cos \theta)}{2} \]

If the vanes are hemispherical \( \theta = 0 \)

\[ \eta_{max} = 1 \text{ or 100%} \]

**Work done by water striking the vanes of a reaction turbine.**

\[ U_1 = \frac{\pi D_1 N}{60} \quad U_1 = R_1 \omega \]

\[ U_2 = \frac{\pi D_2 N}{60} \quad U_2 = R_2 \omega \]

**Angular Momentum Principle:**
Torque = Rate of change of angular momentum

\[ T = (\rho Q) \cdot [V_{w1} R_1 - V_{w2} R_2] \]

1. Inlet tip
2. Outlet tip

\( U_1 \) - Tangential velocity of wheel at inlet
\( U_2 \) - Tangential velocity of wheel at outlet
\( V_1 \) - Absolute velocity of fluid at inlet
\( V_2 \) - Absolute velocity of fluid at outlet
\( V_{w1} \) - Tangential component of absolute velocity at inlet – velocity of wheel at inlet = \( V_1 \cos \alpha_1 \).
\( V_{w2} \) - Tangential component of absolute velocity at outlet – velocity of wheel at outlet = \( V_2 \cos \alpha_2 \).
\( V_{f1} \) - Absolute velocity of flow at inlet
\( V_{f2} \) - Absolute velocity of flow at outlet
\( V_{r1} \) - Relative velocity at inlet
\( V_{r2} \) - Relative velocity at outlet
$\alpha_1$ - Guide angle or guide vane angle at inlet

$\beta_1$ - Vane angle at inlet

$\beta_2$ - Vane angle at outlet

By angular momentum equation

$T = m [V_{w1} R_1 - (V_{w2} R_2)]$

$T = m[V_{w1} R_1 + V_{w2} R_2]$

$T = m[V_{w1} R_1 \pm V_{w2} R_2] \quad \ldots \quad (1)$

Work done/s or power = $T \times$ Angle velocity

Work done/s or power = $T \cdot \omega$

Work done/s or power = $m[V_{w1} R_1 \pm V_{w2} R_2] \cdot \omega$

Work done/s or power = $m [V_{w1} (\omega R_1) \pm V_{w2} (\omega R_2)]$

Work done/s or power = $m [V_{w1} U_1 \pm V_{w2} U_2]$

Work done per unit mass flow rate = $[V_{w1} U_1 \pm V_{w2} U_2]$

Work done per unit weight flow rate = $(1/g)(V_{w1} U_1 \pm V_{w2} U_2)$

Efficiency of the system $\eta = \frac{m(V_{w1} U_1 \pm V_{w2} U_2)}{\frac{1}{2} m V_1^2}$

$\eta = \frac{2(V_{w1} U_1 \pm V_{w2} U_2)}{V_1^2}$
UNIT 6

PELTON WHEEL

HYDRAULIC TURBINES

Introduction:
The device which converts hydraulic energy into mechanical energy or vice versa is known as Hydraulic Machines. The hydraulic machines which convert hydraulic energy into mechanical energy are known as Turbines and that convert mechanical energy into hydraulic energy is known as Pumps. Fig. shows a general layout of a hydroelectric plant.

It consists of the following:

1. A Dam constructed across a river or a channel to store water. The reservoir is also known as Headrace.

2. Pipes of large diameter called Penstocks which carry water under pressure from storage reservoir to the turbines. These pipes are usually made of steel or reinforced concrete.

3. Turbines having different types of vanes or buckets or blades mounted on a wheel called runner.

4. Tailrace which is a channel carrying water away from the turbine after the water...
has worked on the turbines. The water surface in the tailrace is also referred to as tailrace.

**Important Terms:**

**Gross Head (H\(_g\)):** It is the vertical difference between headrace and tailrace.

**Net Head:(H):** Net head or effective head is the actual head available at the inlet of the turbine to work on the turbine.

\[ H = H_g - h_L \]

Where \( h_L \) is the total head loss during the transit of water from the headrace to tailrace which is mainly head loss due to friction, and is given by

\[ h_f = \frac{4fLV^2}{2gd} \]

Where \( f \) is the coefficient of friction of penstock depending on the type of material of penstock

- \( L \) is the total length of penstock
- \( V \) is the mean flow velocity of water through the penstock
- \( D \) is the diameter of penstock and
- \( g \) is the acceleration due to gravity

**TYPES OF EFFICIENCIES**

Depending on the considerations of input and output, the efficiencies can be classified as

- (i) Hydraulic Efficiency
- (ii) Mechanical Efficiency
- (iii) Overall efficiency

(i) **Hydraulic Efficiency:** (\( \eta_h \))

It is the ratio of the power developed by the runner of a turbine to the power supplied at the inlet of a turbine. Since the power supplied is hydraulic, and the probable loss is
between the striking jet and vane it is rightly called hydraulic efficiency.

If R.P. is the Runner Power and W.P. is the Water Power

$$\eta_h = \frac{\text{R.P.}}{\text{W.P.}}$$

(01)

2. Mechanical Efficiency: ($\eta_m$)

It is the ratio of the power available at the shaft to the power developed by the runner of a turbine. This depends on the slips and other mechanical problems that will create a loss of energy between the runner in the annular area between the nozzle and spear, the amount of water reduces as the spear is pushed forward and vice-versa and shaft which is purely mechanical and hence mechanical efficiency.

If S.P. is the Shaft Power

$$\eta_m = \frac{\text{S.P.}}{\text{R.P.}}$$

(02)

3. Overall Efficiency: ($\eta$)

It is the ratio of the power available at the shaft to the power supplied at the inlet of a turbine. As this covers overall problems of losses in energy, it is known as overall efficiency. This depends on both the hydraulic losses and the slips and other mechanical problems that will create a loss of energy between the jet power supplied and the power generated at the shaft available for coupling of the generator.

$$\eta = \frac{\text{S.P.}}{\text{W.P.}}$$

(03)

From Eqs 1, 2 and 3, we have

$$\eta = \eta_h \times \eta_m$$

Classification of Turbines

The hydraulic turbines can be classified based on type of energy at the inlet, direction of flow through the vanes, head available at the inlet, discharge through
the vanes and specific speed. They can be arranged as per the following table:

<table>
<thead>
<tr>
<th>Turbine</th>
<th>Type of energy</th>
<th>Head</th>
<th>Discharge</th>
<th>Direction of flow</th>
<th>Specific Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pelton Wheel</td>
<td>Impulse</td>
<td>High Head &gt; 250m to 1000m</td>
<td>Low</td>
<td>Tangential to runner</td>
<td>Low &lt;35 Single jet</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>35 – 60 Multiple jet</td>
</tr>
<tr>
<td>Francis Turbine</td>
<td>Reaction</td>
<td>Medium Head &gt; 60 m to 150 m</td>
<td>Medium</td>
<td>Radial flow</td>
<td>Medium</td>
</tr>
<tr>
<td></td>
<td>Pressure</td>
<td></td>
<td></td>
<td></td>
<td>60 to 300</td>
</tr>
<tr>
<td>Kaplan Turbine</td>
<td>Pressure</td>
<td>Low Head &lt; 30 m</td>
<td>High</td>
<td>Axial Flow</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>300 to 1000</td>
</tr>
</tbody>
</table>

As can be seen from the above table, any specific type can be explained by selecting the other items in the table along the row.

**PELTON WHEEL OR TURBINE**

Pelton wheel, named after an eminent engineer, is an impulse turbine wherein the flow is tangential to the runner and the available energy at the entrance is completely kinetic energy. Further, it is preferred at a very high head and low discharges with low specific speeds. The pressure available at the inlet and the outlet is atmospheric.

(i) The water jet has to reduce and increase as the spear is brought forward and backward.
The main components of a Pelton turbine are:

(i) **Nozzle and flow regulating arrangement:**

Water is brought to the hydroelectric plant site through large penstocks at the end of which there will be a nozzle, which converts the pressure energy completely into kinetic energy. This will convert the liquid flow into a high-speed jet, which strikes the buckets or vanes mounted on the runner, spear wheel which in turn rotates the runner of the turbine. The amount of water striking the vanes is controlled by the forward and backward motion of the spear. As the water is flowing in the annular area between the annular area between the nozzle opening and the spear, the flow gets reduced as the spear moves forward and vice versa.

2. **Runner with buckets:**

Runner is a circular disk mounted on a shaft on the periphery of which a number of buckets are fixed equally spaced as shown in Fig. The buckets are made of cast-iron cast-steel, bronze or stainless steel depending upon the head at the inlet of the turbine. The water jet strikes the bucket on the splitter of the bucket and gets deflected through (α) 160 - 170°.
Casing:

It is made of cast-iron or fabricated steel plates. The main function of the casing is to prevent splashing of water and to discharge the water into tailrace.

Breaking jet:

Even after the amount of water striking the buckets is completely stopped, the runner goes on rotating for a very long time due to inertia. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of bucket with which the rotation of the runner is reversed. This jet is called as breaking jet.

\[
\begin{align*}
F_x &= \text{rate of change of momentum of the jet along the direction of vane motion} \\
F_x &= (\text{Mass of water/second}) \times \text{change in velocity along the x direction} \\
&= \rho a V_1 [V_{w1} - (-V_{w2})] \\
&= \rho a V_1 [V_{w1} + V_{w2}]
\end{align*}
\]

Work done per second by the jet on the vane is given by the product of Force exerted on
the vane and the distance moved by the vane in one second

\[ W \cdot D. /S = F_x \times u \]
\[ = \rho a V_1 \left[ V_{w1} + V_{w2} \right] u \]

Input to the jet per second = Kinetic energy of the jet per second

\[ \frac{1}{2} \rho a V_1^3 \]

Efficiency of the jet = \( \frac{\text{Output / second}}{\text{Workdone / second}} \)
\[
\eta = \frac{\rho a V_1 \left[ V_{w1} + V_{w2} \right] u}{\frac{1}{2} \rho a V_1^3}
\]
\[
= \frac{2 u \left[ V_1^2 + V_{w1}^2 + V_{w2} \right]}{V_1^2}
\]

From inlet velocity triangle, \( V_{w1} = V_1 \)

Assuming no shock and ignoring frictional losses through the vane, we have \( V_{r1} = V_{r2} = (V_1 - u_1) \)

In case of Pelton wheel, the inlet and outlet are located at the same radial distance from the centre of runner and hence \( u_1 = u_2 = u \)

From outlet velocity triangle, we have \( V_{w2} = V_{r2} \cos \phi - u_2 = (V_1 - u) \cos \phi - u \)

\[ F_x = \rho a V_1 \left[ V_1 + (V_1 - u) \cos \phi - u \right] \]
\[ F_x = \rho a V_1 \left( V_1 - u \right) \left[ 1 + \cos \phi \right] \]

Substituting these values in the above equation for efficiency, we have

\[ \eta = \frac{2 u \left[ V_1 + (V_1 - u) \cos \phi - u \right]}{V_1^2} \]
\[ \eta = \frac{2 u}{V_1^2} \left[ (V_1 - u) + (V_1 - \cos \phi) \right] \]
The above equation gives the efficiency of the jet striking the vane in case of Pelton wheel.

To obtain the maximum efficiency for a given jet velocity and vane angle, from maxima-minima, we have

\[
\frac{d \eta}{du} = 0
\]

\[\Rightarrow \frac{d \eta}{du} = \frac{2u}{V_1} \left(1 + \cos \phi \right) \frac{d}{du} \left( uV_1 - u^2 \right) = 0 \]

\[V_1 - 2u = 0\]

or \[u = \frac{V_1}{2}\]
i.e. When the bucket speed is maintained at half the velocity of the jet, the efficiency of a Pelton wheel will be maximum. Substituting we get,

\[ \eta_{\text{max}} = \left( \frac{2u}{2u} \right)^2 (2u - u)[1 + \cos \phi] \]

\[ \eta_{\text{max}} = \frac{1}{2} [1 + \cos \phi] \]

From the above it can be seen that more the value of \( \cos \phi \), more will be the efficiency. From maximum efficiency, the value of \( \cos \phi \) should be 1 and the value of \( \phi \) should be 0°. This condition makes the jet to completely deviate by 180° and this, forces the jet striking the bucket to strike the successive bucket on the back of it acting like a breaking jet. Hence to avoid this situation, at least a small angle of \( \phi = 5° \) should be provided.
UNIT 8

CENTRIFUGAL PUMPS

Introduction

A pump is a hydraulic machine which converts mechanical energy into hydraulic energy or pressure energy. A centrifugal pump is also known as a Rotodynamic pump or dynamic pressure pump. It works on the principle of centrifugal force. In this type of pump the liquid is subjected to whirling motion by the rotating impeller which is made of a number of backward curved vanes. The liquid enters this impeller at its center or the eye and gets discharged into the casing enclosing the outer edge of the impeller. The rise in the pressure head at any point/outlet of the impeller is Proportional to the square of the tangential velocity of the liquid at that point \( i.e., \frac{au^2}{2g} \). Hence at the outlet of the impeller where the radius is more the rise in pressure head will be more and the liquid will be discharged at the outlet with a high pressure head. Due to this high pressure head, the liquid can be lifted to a higher level. Generally centrifugal pumps are made of the radial flow type only. But there are also axial flow or propeller pumps which are particularly adopted for low heads.

- Advantages of centrifugal pumps:-
  1. Its initial cost is low
  2. Efficiency is high.
  3. Discharge is uniform and continuous
  4. Installation and maintenance is easy.
  5. It can run at high speeds, without the risk of separation of flow

Classification of Centrifugal Pumps

Centrifugal pumps may be classified into the following types

1. According to casing design
   a) Volute pump b) diffuser or turbine pump

2. According to number of impellers
a) Single stage pump  
b) multistage or multi impeller pump

3. According to number of entrances to the Impeller
   a) Single suction pump  
b) Double suction pump

4. According to disposition of shaft
   a) Vertical shaft pump  
b) Horizontal shaft pump

5. According to liquid handled
   a) Semi open impeller  
b) Open impeller pump

6. According to specific speed
   a) Low specific speed or radial flow impeller pump Shrouded impeller
   b) Medium specific speed or mixed flow impeller pump
   c) High specific speed or axial flow type or propeller pump.

7. According to head (H)
   a) Low head if \( H < 15 \)m
   b) Medium head if \( 15 < H < 40 \)m
   c) High head if \( H > 40 \)m

In the case of a volute pump a spiral casing is provided around the impeller. The water which leaves the vanes is directed to flow in the volute chamber circumferentially. The area of the volute chamber gradually increases in the direction flow. Thereby the velocity reduces and hence the pressure increases. As the water reaches the delivery pipe a considerable part of kinetic energy is converted into pressure energy. However, the eddies are not completely avoided, therefore some loss of energy takes place due to the continually increasing quantity of water through the volute chamber. In the case of a diffuser pump the guide wheel containing a series of guide vanes or diffuser is the additional component. The diffuser blades which provides gradually enlarging passages surround the impeller periphery. They serve to augment the process of pressure built up that is normally achieved in the volute casing. Diffuser pumps are also called turbine Multistage pumps and vertical shaft deep-well pumps fall under this category.

Centrifugal pumps can normally develop pressures upto 1000kpa (100m). If higher pressures are required there are three options.

a) Increase of impeller diameter.
b) Increase of Rpm.
c) Use of two or more impellers in series.

The pump looks clumsy in option (a). The impeller material is heavily stressed in option (b). The third choice is the best and is generally adopted, the impellers which are usually of the same size are mounted on the same shaft. The unit is called a multistage pump. It discharges the same quantity of fluid as a single stage pump but the head developed is high. There are centrifugal pumps up to 54 stages. However, generally not more than 10 stages are required. In the case of the double suction impeller, two impellers are set back to back. The two suction eyes together reduce the intake. The two suction eyes together reduce the intake velocity reduce the risk of cavitations. Mixed flow type double suction axial flow pumps besides are capable of developing higher heads. For convenience of operation and maintenance, horizontal shaft settings are the preferred setups for centrifugal pumps. The exceptions are deep-well turbine pumps and axial flow pumps, these have vertical shafts. Restricted space conditions usually require a vertical shaft setting. Centrifugal impellers usually have vanes fitted between the shroudes or plate.

The crown plate has the suction eye and the base plate is mounted on a sleeve which is keyed to the shaft. An impeller without the crown plate is called the non-clog or semi open impeller. In an open impeller both crown plate and the base plate are absent. Only clear liquids, can be safely pumped by a shrouded impeller pump. The semi-open impeller is useful for pumping liquids containing suspended solids, such as sewage, molasses or paper pulp. The open-vane impeller pump is employed for dredging operations in harbors and rivers. Shrouded and semi open impellers may be made of castiron or cast steel. Open vane impellers are usually made of forged steel. If the liquid pumped are corrosive, brass, bronze or gun metal are the best materials for making the impellers.

A radial flow impeller has small specific speeds (300 to 1000) & is suitable for discharging relatively small quantities of flow against high heads. The direction of flow
at exit of the impeller is radial. The mixed flow type of impellers has a high specific speed (2500 to 5000), has large inlet diameter D and impeller width B to handle relatively large discharges against medium heads. The axial flow type or propeller impellers have the highest speed range (5000 to 10,000). They are capable of pumping large discharges against small heads. The specific speed of radial pump will be 10<Ns<80, Axial pump 100<Ns<450, Mixed flow pump 80<Ns<160.

Components of a centrifugal pump

The main components of a centrifugal pump are:

**Impeller** is the rotating component of the pump. It is made up of a series of curved vanes. The impeller is mounted on the shaft connecting an electric motor.

**Casing** is an air tight chamber surrounding the impeller. The shape of the casing is designed in such a way that the kinetic energy of the impeller is gradually changed to potential energy. This is achieved by gradually increasing the area of cross section in the direction of flow.

**Suction pipe** It is the pipe connecting the pump to the sump, from where the liquid has to be lifted up.

**Foot valve with strainer** the foot valve is a non-return valve which permits the flow of the liquid from the sump towards the pump. In other words the foot valve opens only in the upward direction. The strainer is a mesh surrounding the valve, it prevents the entry of debris and silt into the pump.

**Delivery pipe** is a pipe connected to the pump to the overhead tank.

**Delivery valve** is a valve which can regulate the flow of liquid from the pump.

**Priming of a centrifugal pump**

Priming is the process of filling the suction pipe, casing of the pump and the delivery pipe upto the delivery valve with the liquid to be pumped. If priming is not done the pump cannot deliver the liquid due to the fact that the head generated by the Impeller will be in terms of meters of air which will be very small (because specific weight of air is very much smaller than that of water).
Priming of a centrifugal pump can be done by any one of the following methods:

i) Priming with suction/vacuum pump.
ii) Priming with a jet pump.
iii) Priming with separator.
iv) Automatic or self priming.

**Heads on a centrifugal pump:**

**Suction head (hs):** It is the vertical distance between the liquid level in the sump and the center line of the pump. It is expressed as meters.

**Delivery head (hd):** It is the vertical distance between the centre line of the pump and the liquid level in the overhead tank or the supply point. It is expressed in meters.

**Static head (Hs):** It is the vertical difference between the liquid levels in the overhead tank and the sump, when the pump is not working. It is expressed as meters.

Therefore, \( HS = (hs + hd) \)

**Friction head (hf):** It is the sum of the head loss due to the friction in the suction and delivery pipes. The friction loss in both the pipes is calculated using the Darcy’s equation, \( hf = (fL V^2/2gD) \).

**Total head (H):** It is the sum of the static head Hs, friction head (hf) and the velocity head in the delivery pipe \( (V_d^2/2g) \). Where, \( V_d \) = velocity in the delivery pipe.

\[
H = h_s + h_d + hf + \frac{V_d^2}{2g} \quad (I)
\]

**Manometric head (Hm):** It is the total head developed by the pump. This head is slightly less than the head generated by the impeller due to some losses in the pump.

\[
H_m = H + \frac{V_s^2}{2g} - \frac{V_d^2}{2g}
\]

**Working of a centrifugal pump:**

A centrifugal pump works on the principal that when a certain mass of fluid is rotated by an external source, it is thrown away from the central axis of rotation and a centrifugal head is impressed which enables it to rise to a higher level. Working operation of a centrifugal pump is explained in the following steps.

1) Close the delivery valve and prime the pump.
2) Start the motor connected to the pump shaft, this causes an increase in the impeller pressure.

3) Open the delivery valve gradually, so that the liquid starts flowing into the delivery pipe.

4) A partial vacuum is created at the eye of the centrifugal action, the liquid rushed from the sump to the pump due to pressure difference at the two ends of the suction pipe.

5) As the impeller continues to run, move & more liquid is made available to the pump at its eye. Therefore impeller increases the energy of the liquid and delivers it to the reservoir.

6) While stopping the pump, the delivery valve should be closed first, otherwise there may be back flow from the reservoir. It may be noted that a uniform velocity of flow is maintained in the delivery pipe. This is due to the special design of the casing. As the flow proceeds from the tongue of the casing to the delivery pipe, the area of the casing increases. There is a corresponding change in the quantity of the liquid from the impeller. Thus a uniform flow occurs in the delivery pipe.

Operation difficulties in centrifugal pumps

a) Pump fails to pump the fluid.

<table>
<thead>
<tr>
<th>Cause</th>
<th>Remedial Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Improper priming due to leakage of foot valve or incomplete filling.</td>
<td>Repair or replace the foot valve, prime completely.</td>
</tr>
<tr>
<td>2) Head more than design head</td>
<td>Reduce the head or change the pump</td>
</tr>
<tr>
<td>3) Clogging of impeller, suction pipe or strainer</td>
<td>Clean the suspected part</td>
</tr>
<tr>
<td>4) Speed more than design speed</td>
<td>Connect another prime mover of higher speed</td>
</tr>
<tr>
<td>5) Direction of rotation of impeller is wrong</td>
<td>Change the direction.</td>
</tr>
<tr>
<td>6) Suction lift may be excessive</td>
<td>Reduce the height of pump above the sump</td>
</tr>
</tbody>
</table>
B) Pump does not give the required capacity

<table>
<thead>
<tr>
<th>a) Leakage of air through the suction pipe or through the gland packing</th>
<th>Stop the leakage</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Damage to some parts of the pump by wear &amp; tear</td>
<td>Replace the damaged parts</td>
</tr>
<tr>
<td>c) Clogging of impeller passages</td>
<td>Clean the impeller</td>
</tr>
</tbody>
</table>

C) Pump has poor efficiency

<table>
<thead>
<tr>
<th>a) Higher than design speed</th>
<th>Reduce the speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Low head &amp; higher discharge</td>
<td>Reduce the discharge</td>
</tr>
<tr>
<td>c) Impeller touching, the casing or improper alignment of shaft</td>
<td>Carry out the necessary repair</td>
</tr>
</tbody>
</table>

D) Pump stops working

<table>
<thead>
<tr>
<th>a) Air entry into suction pipe</th>
<th>Stop the pump, plug the leakage, Re prime and start</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Suction lift is high</td>
<td>Reduce the suction lift</td>
</tr>
</tbody>
</table>

Efficiencies of centrifugal pump

Manometric efficiency (\(\eta_{\text{mano}}\)): it is the ratio of the manometric head to the head actually generated by the impeller

\[
\eta_{\text{mano}} = \left( \frac{H_m}{Vw_ju_2/g} \right) = \left( \frac{gH_m}{Vw_ju_2} \right)
\]

Mechanical efficiency (\(\eta_{\text{mech}}\)): It is the ratio of the impeller power to the power of the motor or the prime mover.

\[
\eta_{\text{mech}} = \frac{\text{impeller power}}{\text{motor power}}
\]

Overall efficiency (\(\eta_0\)): It is the ratio of the work done by the pump in lifting water against gravity and friction in the pipes to the energy supplied by the motor.
Velocity Triangles of a Centrifugal Pump

Figure shows the inlet and outlet velocity triangles for a centrifugal pump. It may be noted that the inlet velocity triangle is radial, (velocity of whirl is zero at inlet or $V_{w1} = 0$).

Depending on the geometry of the blade at outlet it can be:
- Forward: if the blade angle $< 90^\circ$,
- Radial if $\theta = 90^\circ$,
- Backward if $\theta > 90^\circ$.

Work done by the impeller of a centrifugal pump:

Figure shows the velocity triangles at the inlet and outlet tips of a vane fixed to the impeller.

Let $N =$ speed of the impeller in RPM

$D =$ Diameter of the impeller at inlet

$U_1 =$ Tangential velocity of the impeller at inlet $\pi D1N/60$

$U_2 =$ tangential velocity of the impeller at outlet $\pi D2N/60$

$V_1 =$ absolute velocity of the liquid at inlet

$V_2 =$ absolute velocity of the liquid at outlet.

$V_{f1} \ & \ V_{f2}$ are the velocities of flow at inlet and outlet.

$V_{r1} \ & \ V_{r2}$ Relative velocities at inlet and outlet

$V_{w2}$ whirl velocity at outlet

$\alpha =$ angle made by $V_1$ with respect to the motion of the vane

$\theta =$ blade angle at inlet

$\phi =$ blade angle at outlet

For a series of curved vanes the force exerted can be determined using the impulse momentum equation $\text{Work} = \text{force} \times \text{distance}$. 

\[ \therefore \ n_r = \begin{cases} \text{work done against gravity friction} \\ \text{power of the prime mover or motor} \end{cases} \]
similarly the work done/sec/unit weight of the liquid striking the vane = $$\frac{1}{g} (Vw_{1}u_{2} - Vw_{2}u_{1})$$

But for a centrifugal pump $V_{\omega 1} = 0$

Work done/sec/unit weight = 
Work done/sec/unit weight

And the work done/sec = (4)

Where $Q =$ volume of liquid flowing per second = Area x velocity of flow = $Q \pi D_{2}B_{2}V_{f2}$  – (5)

In eq (5), $B_{2}$ is the width of the impeller at the outlet.

Performance of centrifugal pumps:

Generally a centrifugal pump is worked under its maximum efficiency conditions, however when the pump is run at conditions other than this it performs differently. In order to predict the behaviour of the pump under varying conditions of speed, discharge and head, full scale tests are usually performed. The results of these tests are plotted in the form of characteristic curves. These curves are very useful for predicting the performance of pumps under different conditions of speed, discharge and head.

Following four types of characteristic curves are usually prepared for a centrifugal pump.

a. Main characteristic.

b. Operating characteristics

c. Constant efficiency or Muschel characteristic.

d. Constant head an constant discharge curves.

Main Characteristic: the pump is operated a particular constant speed, discharge is varied by adjusting the delivery valve. Manometric head $H_{m}$ and the shaft power $P$ are measured for each discharge. The overall efficiency is then calculated. The curves are plotted between $H_{m}$ & $Q$, $P$ & $Q$, & $Q$. A set of similar curves are plotted by running the pump at different speeds. They will be as shown.

b. Operating characteristic: The curves are obtained by running the pump at the design speed, which is also the driving speed of the motor. The design discharge and head are
obtained from the corresponding Curves, where the efficiency is maximum as shown.

c. **Constant efficiency curves:** The constant efficiency curves are obtained from the main characteristic curves. The line of maximum efficiency is obtained by joining the points of the maximum curvature of the constant efficiency lines. These curves are useful in determining the range of operation of a pump.

d. **Constant head and constant discharge curves:** If the pump has a variable speed, the plots between Q and N and that between Hm and N may be obtained by varying the speed. In the first case Hm is kept constant & in the second Q is kept constant.
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